

# Math SL PROBLEM SET 56

## Section A (Short Answer)

1. **(SP5.1, SP5.6 - R) (CA)** Consider the given cumulative frequency table: **(Oxford 8.3, p260)**

$x$	frequency	cumulative frequency
2	4	4
4	6	10
6	$k$	20
8	9	29
10	7	36

- a. Find the value of  $k$ .
- b. Find the mean and the variance.
- c. Determine  $P(x > 8 \mid x > 4)$ .
2. **(A1.3 - R) (CA)** The number of people participating in Maadi's annual Sum Fun Run has been monitored and it has been noticed that the number of participants has increased by 12.5% each year. In the first Sum Fun Run (held in 1990), forty people participated. **(Cirrito 8.2, p252)**
- a. Write down the number of people who participated in fun run in 1991.
- b. Find the number of people that participated in the fun run in 2018.
- c. In what year the total number of participants first exceed 15,000?
3. **(T3.6 - R) (CA)** Given the triangle  $FGH$ , where  $FG = 13$ ,  $GH = 10$  and angle  $HFG = 35^\circ$ . **(Cirrito 9.5.2, p294)**
- a. Find the two possible values of angle  $FHG$ .
- b. Hence, find angle  $FGH$ , given that it is acute.
4. **(A1.3 - R) (CA)** When the expression  $(\frac{1}{4}x + 2a)^7$  is expanded, one of the terms in the expansion is  $140x^3$ . Find the possible values of  $a$ . **(Cirrito 4.1.2, p100)**
5. **(A1.2 F2.7 - R) (CA)** For the function  $g(x) = \log_3 \frac{x}{2} + 2 \log_3 4 - \log_3 2$ , for  $x > 0$ . **(Cirrito 7.4, p221; Cirrito 5.4, p148)**
- a. Show that equation for  $g(x)$  can be simplified to  $g(x) = \log_3(4x)$ .
- b. The inverse of  $g$  can be written in the form of  $g^{-1}(x) = ab^x$ . Determine the values of  $a$  and the value of  $b$ .
- c. Determine the solution(s) to the equation  $g^{-1}(x) = g(x)$ .
6. **(A1.3 - R) (CA)** Write  $\log_3 \frac{x^2}{27}$  in the form of  $a + b \log_3 x$  where  $a$  and  $b$  are integers. **(Cirrito 7.4, p221)**

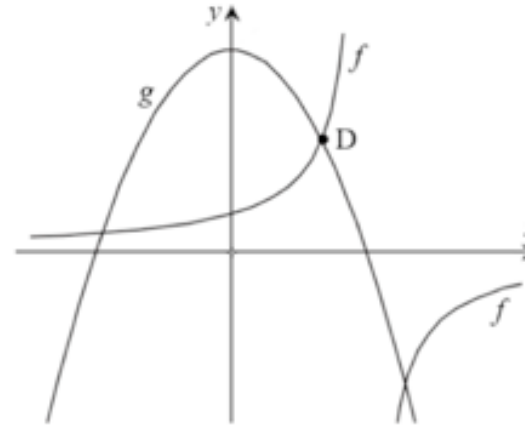
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## Section B (Extended Response/Investigation)

7. **(C6.1, C6.4 - R) (CA)** Given the functions  $f(x) = \frac{10}{6-x}$  and  $g(x) = 9 - \frac{1}{4}x^2$ . Sketches of these functions are shown below. The graphs of  $f(x)$  and  $g(x)$  intersect at three points, one of which is at  $D(a,b)$ , where  $a, b > 0$ . **(Cirrito 20.1, p643; Cirrito 22.5, p748)**
- Find the value of  $a$  and the value of  $b$ .
  - Evaluate  $g'(a)$ .

Let  $L$  be the normal to the graph of  $g$  at  $D$ .

- Find the equation of  $L$ , giving your answer in the form of  $y = mx + c$ .
- Sketch the function  $g(x)$  as well as the line  $L$ . Label the key features in your sketch.
- Given the region enclosed by the graph of  $g(x)$  and the line  $L$ . Shade this region on your sketch and then find the area of this shaded region.



8. **(T3.1, T3.6 - R) (CA)** The diagram shows a circle with centre  $O$  and a radius of 6 cm. The points  $A$ ,  $B$ , and  $C$  lie on the circle. The point  $D$  is outside the circle and lies on  $OC$ . Angle  $AOC = 1.2$  radians and angle  $ADO = 0.25$  radians. **(Cirrito 9.7, p309)**
- Find the area of the sector  $OABC$ .
  - Find the area of the shaded region bounded by the chord  $AC$  and the arc  $ABC$ .
  - Determine the length of  $OD$ .
  - Hence or otherwise, the area of region  $ABCD$ .

