

Math SL PROBLEM SET 54

Section A (Short Answer)

1. **(T3.6 - R) (CA)** Three towns are positioned so that Blacktown is 15 km north of Acton and Capetown is 32 km N45°E (on a bearing of 45°) of Acton.
- Draw a triangle representing the locations of the three towns and label all information.
 - Calculate the distance between Blacktown and Capetown.
 - Determine the measure of angle BCA.

An airplane is 3.0 km directly above Blacktown at point P. The points A, P, C form a triangle.

- Determine how far the plane is from Acton.
 - Calculate the size of angle APC.
2. **(T3.5 - R) (CI)** Solve the following trigonometric equations:
- $1 + \sqrt{3} \tan 2(x - \frac{\pi}{3}) = 0$ on the domain of $0 < x < \pi$.
 - $2 \cos(\frac{x}{2}) - 1 = 0$ on the domain of $-\pi < x < \pi$.
3. **(V4.3, V4.4 - R) (CA)** Given the lines: $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ -4 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$
- Do these two lines intersect?
 - If so, where?
 - Find the measure of the acute angle between the direction vectors of these two lines.
4. **(F2.7, C6.2, C6.4 - R) (CI)** Let the function f be defined by $f(x) = x - 3 + \frac{4}{x^2}$, where $x \neq 0$.
- Find the values for x for which $f(x) \geq 0$.
 - Find the values for x for which $\frac{df}{dx} \geq 0$.
 - If the function f represents a **derivative** of $g(x)$, determine the equation for $g(x)$.
5. **(SP5.7 - R) (CA)** A discrete random variable, X , may take on the values 0, 1 or 2. The probability distribution of X is defined by $P(X=x) = \frac{k}{x!}$. Find
- the value of k
 - the mean and variance of X .
6. **(SP5.6 - R) (CA)** Mr D. walks home after teaching and takes either Road 253 or Road 206 to get home. He varies his route so that he takes Road 253 two-thirds of the time. If he walks along Road 253, he arrives home before 6:00 pm 90% of the time. If he takes Road 206, he gets home by 6:00 pm only 60% of the time. What is the probability that:
- he gets home after 6:00 pm?
 - he travelled along Road 206, if he gets home before 6:00 pm?

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Section B (Extended Response/Investigation)

7. **(A1.1 - R) (CA)** Three companies A, B and C are competitors and are comparing sales figures for the period July 1st, 2013 to June 30th, 2014 (so, one year).

Company A had sales of \$35,400 in July of 2013 and increased sales by an average of 3% per month over this period.

Company B had sales of \$32,000 in July of 2013 and increased sales by an average of \$1859 per month over this period.

Company C had sales of \$48,000 in July of 2013 and unfortunately, their sales decreased by an average of 8% per month over this period.

- a. Calculate to the nearest hundred dollars:
 - i. The sales figures for each of the companies in June of 2014.
 - ii. The **total** sales figures for each of the companies for this 12 month period.
 - b. Calculate, correct to one decimal place, the average percentage increase in sales per month over this one year period for Company B.
 - c. Assuming the sales trends **continue** beyond June 2014,
 - i. calculate how many months from July 2013 it would take Company A to reach a **total sales** of \$600,000.
 - ii. will Company C ever reach a **total sales** target of \$600,000? Give reasons.
8. **(T3.4 - R) (CA)** The depth of the water at a certain point on the coast, at time t hours after 12:00 (noon) on any given day is modeled by $h(t) = 2.5 + \frac{1}{2} \cos \frac{4\pi}{25} (t + 2)$, $t \geq 0$
- a. What is the depth of the water at noon?
 - b. What is the depth of the water at (i) high tide? (ii) low tide?
 - c. At what time on the afternoon of the first day will low tide occur?
 - d. Sketch a graph of $h(t)$ for $0 \leq t \leq 12.5$
 - e. At what rate is the depth of the water changing at 3:00 pm on the first day? Show/explain your answer.
 - f. At what times in that first day will the water depth be at least 2.25 m?