Math SL PROBLEM SET 52

Section A (Short Answer) (NOTE: All Qs are CA)

Q1

[Maximum mark: 7]

In an arithmetic sequence, the third term is 41 and the ninth term is 23.

(a) Find the common difference. [2]

(b) Find the first term. [2]

(c) Find the smallest value of n such that $s_n < 0$ [3]

Q2

[Maximum mark: 6]

The maximum temperature T, in degrees Celsius, on six randomly selected days is shown in the following table. The table also shows the number of soda cans purchased, N, from a vending machine.

Maximum temperature (T)	5	6	18	32	29	12
Number of soda cans (N)	26	28	37	41	48	29

The relationship between the variables can be modelled by the regression equation $N = aT + b$.

(a) Find the value of r, the correlation coefficient. [2]

(b) (i) Find the value of a and b.

(ii) Hence, use the regression equation to estimate the number of soda cans purchased on a day when the maximum temperature is 23°C. [4]

Q3

[Maximum mark: 7]

Let $f(x) = x^3 + 1$ and $g(x) = x - 2$, for $x \in \mathbb{R}$.

(a) Find $f(2)$. [2]

(b) Find $f^{-1}(x)$. [2]

(c) Solve $(f \circ g)(x) = 0$. [3]

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Q4

[Maximum mark: 6]

The values of the functions f and g and their derivatives for $x = 3$ and $x = 7$ are shown in the following table.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	3	−2	−8	2
7	5	6	1	3

Let $h(x) = f(x)g(x)$.

(a) Find $h(3)$. [2]

(b) Find the equation of the normal to h when $x = 7$. [4]

Q5

[Maximum mark: 6]

A bag contains 8 blue marbles, 12 green marbles and m red marbles. A marble is selected at random and replaced. This is performed three times.

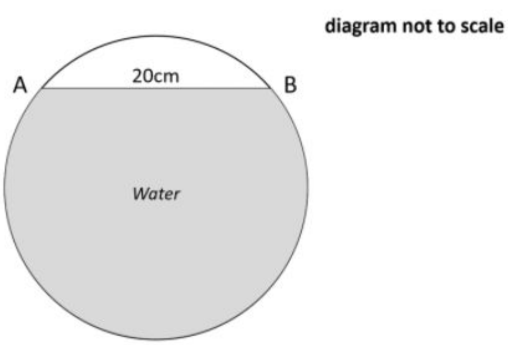
(a) Write down the probability that the first marble selected is blue. [1]

(b) Let X be the number of blue marbles selected. Find the smallest value of m for which $\mathrm{Var}(X) < 0.5$. [5]

Q6

[Maximum mark: 7]

The diagram below shows a cylindrical pipe, 80cm in length, carrying water. The pipe has a radius of 15cm.



The pipe is not at full capacity, such that the chord length of the water level [AB] is 20cm.

Find the volume of water in the pipe.

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Q7

[Maximum mark: 7]

Let $f(x) = (x^2 + k)^5$.

In the expansion of the derivate, $f'(x)$, the coefficient of the term in x^5 is 960. Find the possible values for k.

Section B (Extended Response) (ALL Qs are CA)

Q8

[Maximum mark: 15]

Note: In this question, distance is in meters and time is in seconds.

A particle P moves in a straight line for six seconds. Its acceleration during this period is given by $a(t) = -2t^2 + 13t - 15$, for $0 \leq t \leq 6$.

(a) Write down the values of t when the particle's acceleration is zero. [2]

(b) Hence or otherwise, find all possible values of t for which the velocity of P is increasing. [2]

The particle has an initial velocity of 7ms^{-1}

(c) Find an expression for the velocity of P at time t. [6]

(d) Find the total distance travelled by P when its velocity is decreasing. [5]

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Q18 – Distributions

[Maximum mark: 13]

Tom enters a tennis competition where he plays three matches every day.
The results of each match are independent of each other.

Let A be the number of matches Tom wins on any given day of the competition.
The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$P(A=a)$	0.15	0.2	p	0.25

(a) Find the value of p. [2]

(b) (i) A day is chosen at random. Write down the probability that Tom wins every match he plays.

(ii) The competition goes for four days. Find the probability that Tom wins every match on exactly three of these days. [3]

Clare enters the same tennis competition. Let B be the number of matches Clare wins on any given day of the competition. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$P(B=b)$	0.05	0.1	0.35	0.5

(c) Find $E(B)$. [2]

On the final day of the competition, both Tom and Clare play their three respective matches and their results are independent. The number of wins Tom and Clare obtain are then added together to form a total out of six.

(d) (i) Find the probability that they win more than four matches combined.

(ii) Given that they win more than four matches combined, find the probability that Clare won all her matches. [6]

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Q10

[Maximum mark: X]

Let $f(x) = \ln x$ and $g(x) = 2 + 3\ln(x-1)$, for $x > 1$.

The graph of g can be obtained from the graph of f by two transformations:

a vertical stretch of scale factor q

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$

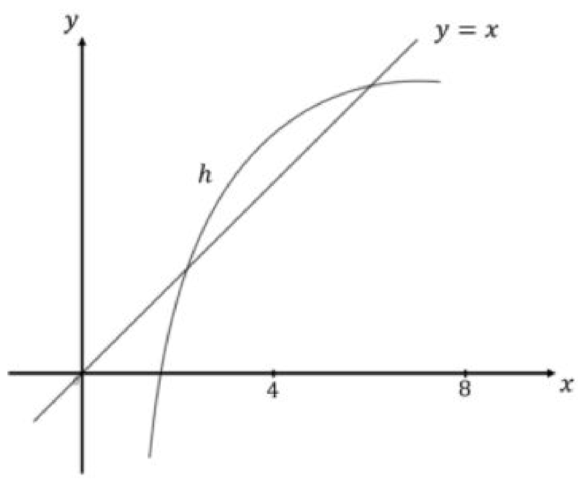
(a) Write down the value of

(i) q;

(ii) h;

(iii) k; [3]

Let $h(x) = g(x) \times \cos(0.1x)$, for $1 < x < 8$. The following diagram shows the graph of h and the line $y = x$.



The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 2.02 and 5.57, correct to three significant figures.

(b) (i) Find $\int_{2.02}^{5.57} (h(x) - x)dx$.

(ii) Hence, find the area of the region enclosed by the graphs of h and h^{-1}. [6]

(c) Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $Q(x, y)$ on the graph of h where d is a maximum. Find the coordinates of Q, where $2.02 < x < 5.57$. [6]