

Math SL PROBLEM SET 51

Section A (Short Answer) (NOTE: All Qs are CI)

- Events A and B are independent with $P(A \cap B) = P(A \cap B^c) = 0.3$
 - Find $P(A)$.
 - Find $P(A \cup B)$.
- Let $\sin(\theta) = \frac{2}{3}$, where θ is obtuse.
 - Find $\cos(\theta)$.
 - Find $\cos(2\theta)$.
- Let $f(x) = e^x - 2$ and $g(x) = 3 + \frac{2}{x}$, for $x \in \mathbf{R}$.
 - Find $(g \circ f)(x)$.
 - Find the exact value of the vertical asymptote of $(g \circ f)(x)$.
 - Find $\lim_{x \rightarrow -\infty} (g \circ f)(x)$.
- Consider the function $f(x) = \ln(2x - 1)$. Let point A be a point on the curve where $x = 3$.
 - Determine the gradient (slope) of the curve at A.
 - The normal to the curve at A cuts the x -axis at P. Find the coordinates of P.
- The first three terms of a geometric sequence are $\ln(x^9)$, $\ln(x^3)$, $\ln(x)$ for $x > 0$.
 - Find the common ratio.
 - Solve the equation $27 = \sum_{k=1}^{\infty} 3^{3-k} \ln x$.
- Solve $\log_{\sqrt{3}}(\sin x) - \log_{\sqrt{3}}(\cos x) = 1$ for $0 < x < \frac{\pi}{2}$.
- The line L passes through the points P(5,0) and Q(0,2). The origin is at O. The point S(3 - 4x, x) is on L, and the line OS is perpendicular to L.
 - Write down the vectors $\langle PQ \rangle$ and $\langle OS \rangle$.
 - Find the coordinates of the point S.

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Section B (Extended Response/Investigation)

8. Let $f(x) = 2x^3 + 3x^2 - 12x + 10$ for $x \in \mathbf{R}$.
- Find the x -coordinates of the extrema.
 - Use the second derivative to classify the extrema as maximums, minimums or neither.
 - (CA) Evaluate $\int_a^b f(x)dx$ where a and b are the x -coordinates of the extrema from Q(a)
(NOTE: $b > a$).
9. A quadratic function f can be written in the form $f(x) = a(x - 1)(x - b)$. The graph of f has an axis of symmetry at $x = 2$ and a y -intercept at $(0, -3)$.
- Find the value of b .
 - Find the value of a .
 - The line $y = kx + 6$ is a tangent to the curve of f . Find the values of k .
 - The quadratic function, $f(x)$, is the derivative of some other function, $F(x)$. If $F(3) = 2$, find the equation of $F(x)$.
10. The following table shows the probability distribution of a discrete random B, in terms of an angle θ .

b	0	1
$P(B = b)$	$2 \cos(2\theta)$	$\cos(\theta)$

- Show that $\cos \theta = \frac{3}{4}$.
- Find $\tan \theta$, given that $\tan \theta > 0$.

Let $y = \frac{1}{\cos x}$ for $0 < x < \frac{\pi}{2}$.

- (CA) Sketch a graph of $y = \frac{1}{\cos x}$ between $0 < x < \frac{\pi}{2}$, labelling the intercepts and extrema.
- (CA) Hence, find the area that lies between the x -axis and the curve between the given x values of $x = \theta$ and $x = \frac{\pi}{4}$. Shade this area in your sketch from Q(c).