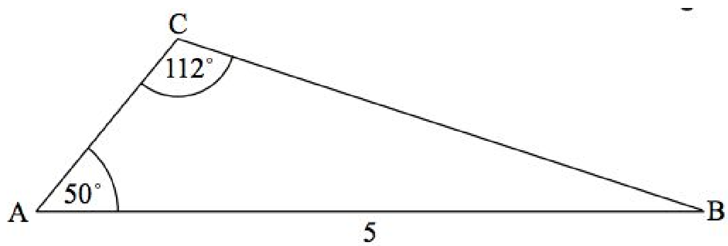
Math SL PROBLEM SET 50

Section A (Short Answer) (NOTE: ALL Qs are CA)

1. The following diagram shows triangle ABC.

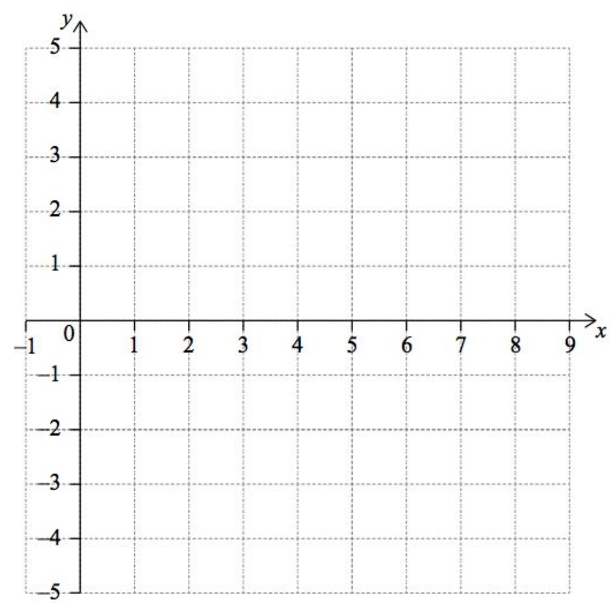
    a. Find BC.
    b. Find the area of triangle ABC.



$AB = 5\,\text{cm}$, $C\hat{A}B = 50°$ and $A\hat{C}B = 112°$

2. Let $f(x) = \frac{6x^2-4}{e^x}$, for $0 \le x \le 7$.

    a. Find the x-intercept of the graph of f.
    b. The graph of f has a maximum at the point A. Write down the coordinates of A.
    c. On the following grid, sketch the graph of f.



3. Let $\overrightarrow{AB} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$.

    a. Find $\left|\overrightarrow{AB}\right|$
    b. Let $\overrightarrow{AC} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$. Find $B\widehat{A}C$.

4. A discrete random variable X has the following probability distribution.

X	0	1	2	3
$P(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

    a. Find the value of k.
    b. Write down $P(X=2)$.
    c. Find $P(X=2 \mid X>0)$.

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5. Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \boldsymbol{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

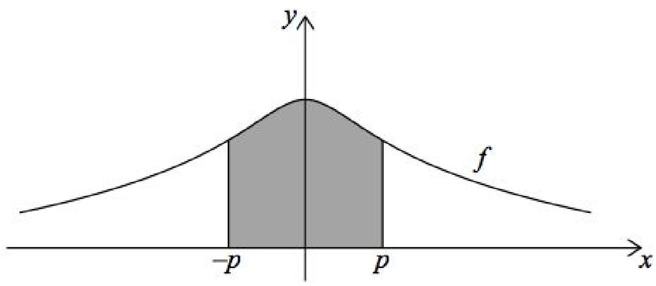
   a. Find the value of p
      i. as an exact value;
      ii. and also write down its value, rounded to the nearest hundredths.

   b. Determine the equation of the derivative of $f(x) = 6 - \ln(x^2 + 2)$.

   c. Hence or otherwise, find the equation of the line normal to f at the point $(p,4)$.

   The following diagram shows part of the graph of f.

   d. The shaded region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is shaded. Find the area of the shaded region shown.



6. In the expansion of $ax^3(2 + ax)^{11}$, the coefficient of the term in x^5 is 11880. Find the value of a.

7. John likes to play two games, A and B.

   For game A, the probability John wins is 0.8. He plays game A five times.

   a. Find the probability he wins exactly two games.

   For game B, the probability John wins is k. He plays game B six times.

   b. Write down an expression, in terms of k, for the probability that he wins exactly four games.
   c. Hence, find the value of k, given that the probability he wins exactly four games is 0.2.

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Section B (Extended Response) (NOTE: ALL Qs are CA)

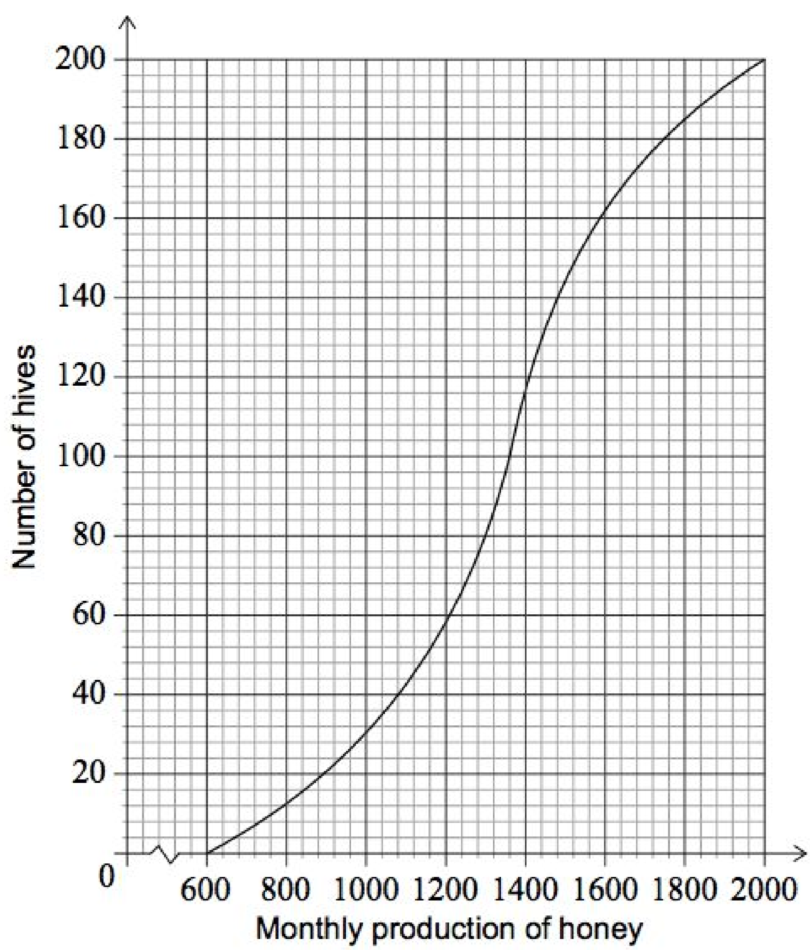
8. Adam is a beekeeper who collected data about monthly honey production in his bee hives. The data for six of his hives is shown in the following table.

Number of bees (N)	190	220	250	285	305	320
Monthly honey production in grams (P)	900	1100	1200	1500	1700	1800

The relationship between the variables is modelled by the regression line, $P = aN + b$.

a. Write down the value of a and of b.
b. Use this regression line to estimate the monthly honey production from a hive that has 270 bees.

Adam has 200 hives in total. He collects data on the monthly honey production of all the hives. This data is shown in the following cumulative frequency graph.



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Adam's hives are labelled as low, regular or high production, as defined in the following table

Type of hive	low	regular	high
Monthly honey production in grams (*P*)	$P \leq 1080$	$1080 < P \leq k$	$P > k$

c. Write down the number of low production hives.

Adam knows that 128 of his hives have a regular production.

d. Find
   i. the value of k ;
   ii. the number of hives that have a high production.

e. Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives.

9. A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a(t) = 3t^2 - 14t + 8$, for $0 \leq t \leq 5$.

Note: In this question, distance is in metres and time is in seconds.

a. Write down the values of t when $a(t) = 0$.
b. Hence or otherwise, find all possible values of t for which the velocity of P is decreasing.

When $t = 0$, the velocity of P is 3 ms^{-1} .

c. Find an expression for the velocity of P at time t.
d. Find the total distance travelled by P when its velocity is increasing.

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10. Let $f(x) = x + a\sin\left(x - \frac{\pi}{2}\right) + a$, for $x \geq 0$.

Note: In this question, distance is in millimetres.

a. Show that $f(2\pi) = 2\pi$.

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k \in \boldsymbol{N}$. A straight line $\boldsymbol{L}$ passes through all the points P_k .

b. (i) Find the coordinates of P_0 and of P_1 .
(ii) Find the equation of $\boldsymbol{L}$.

c. Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π .

Diagram 1

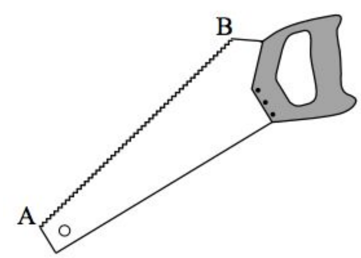
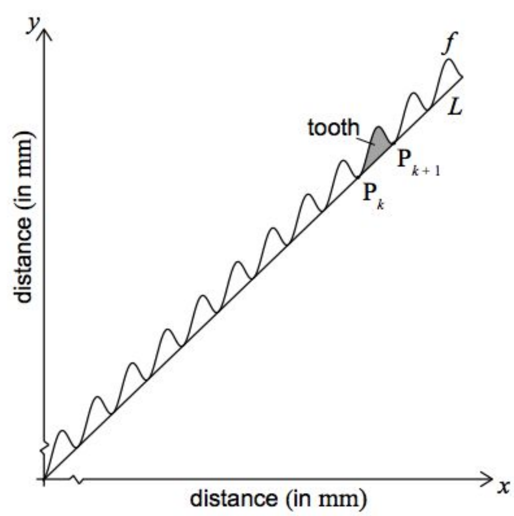


Diagram 1 shows a saw. The length of the toothed edge is the distance AB

The toothed edge of the saw can be modelled using the graph of f and the line $\boldsymbol{L}$. Diagram 2 represents this model.

Diagram 2



The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of $\boldsymbol{f}$ and the line $\boldsymbol{L}$, between P_k and P_{k+1} .

d. A saw has a toothed edge which is 300mm long. Find the number of complete teeth on this saw.