Math SL PROBLEM SET 49

Section A (Short Answer) (NOTE: ALL Qs are CI)

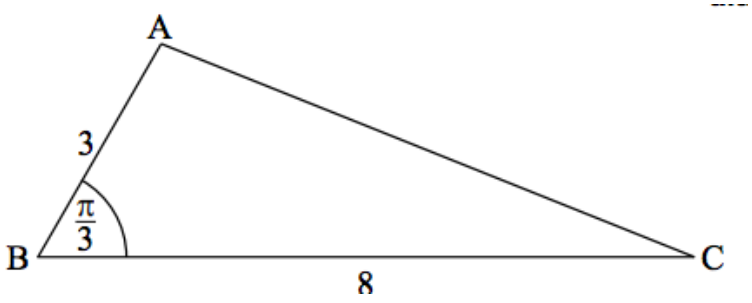
1. A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

    a. Complete a tree diagram.
    b. Find the probability that exactly one of the selected balls is green.

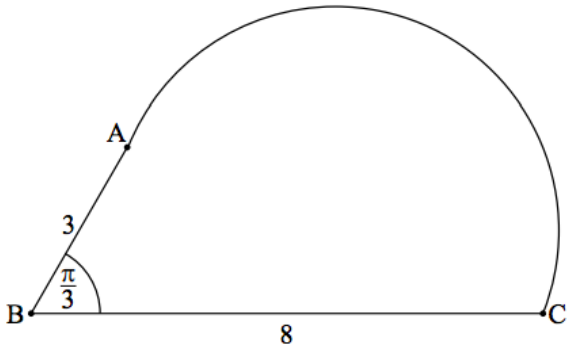
2. In an arithmetic sequence, the first term is 8 and the second term is 5.

    a. Find the common difference.
    b. Find the tenth term.
    c. Find the sum of the first ten terms.

3. The following diagram shows triangle ABC, with AB = 3 cm, BC = 8 cm, and $\angle ABC = \frac{\pi}{3}$.



    a. Show that AC = 7 cm.
    b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. Find the exact perimeter of this shape.



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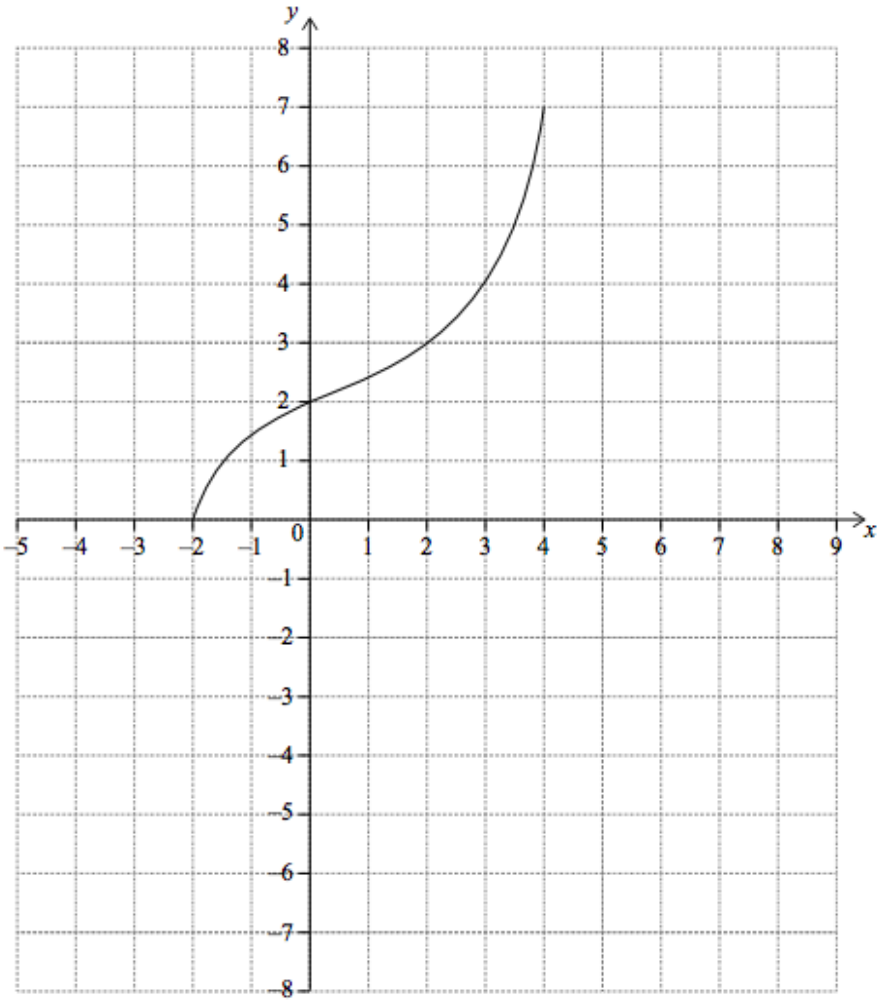
4. Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \boldsymbol{R}$, where b is a constant.

   a. Find $gof(x)$.
   b. Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b.

5. Consider $f(x) = \log_k (6x - 3x^2)$, for $0 < x < 2$, where $k > 0$. The equation $f(x) = 2$ has exactly one solution. Find the value of k.

6. The diagram below shows the graph of a function f, with domain $-2 \leq x \leq 4$. The points (-2 , 0) and (4 , 7) lie on the graph of f.

   a. Write down the range of f.
   b. Write down: (*i*) $f(2)$; (*ii*) $f^{-1}(2)$.
   c. On the grid, sketch the graph of f^{-1}.



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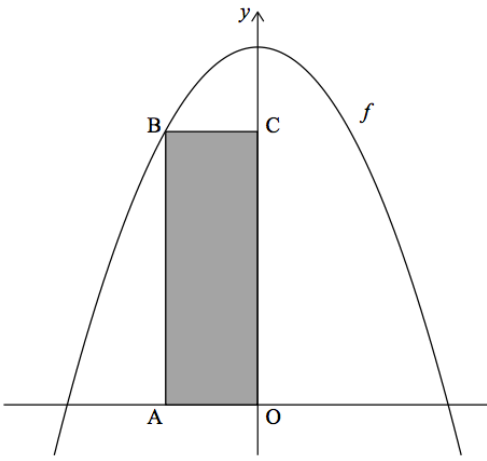
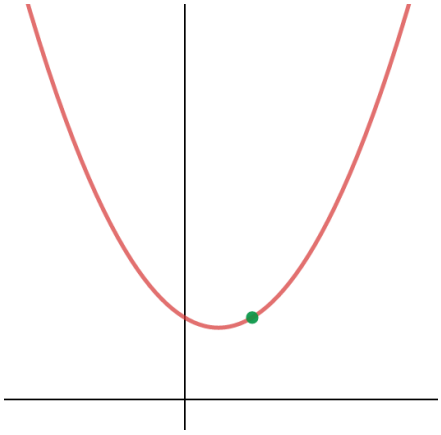
7. Let $f(x) = 15 - x^2$, for $x \in \boldsymbol{R}$. The diagram shows part of the graph of f and the rectangle $OABC$, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.



   a. Show that the area of the rectangle can be expressed as $A(x) = 15x - x^3$.
   b. Hence, or otherwise, find the x-coordinate of A that gives the **maximum area** of $OABC$.

Section B (Extended Response/Investigation) (NOTE: ALL Qs are CA)

8. Let $f(x) = x^2 - x + 2$, for $x \in \boldsymbol{R}$. The following diagram shows part of the graph of f. The point P is located at $x = 1$.

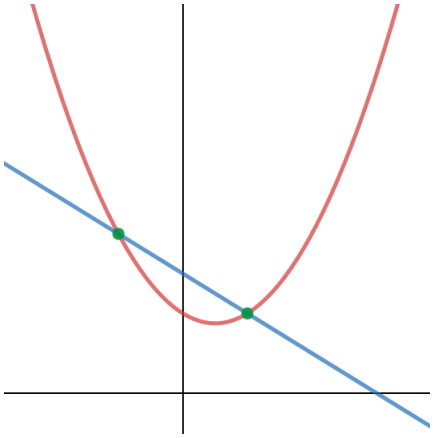


   a. Show that $f'(1) = 1$.

The line $\boldsymbol{L}$ is the normal to the graph of f at P.

   b. Find the equation of $\boldsymbol{L}$ in the form $y = ax + b$.

The line $\boldsymbol{L}$ intersects the graph of f at another point Q, as shown in the following diagram.



   c. Find the x-coordinate of Q.
   d. **(CA - for now)** Find the area of the region enclosed by the graph of f and the line $\boldsymbol{L}$.

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9. A line ***L*** passes through points $A(-3, 4, 2)$ and $B(-1, 3, 3)$.

   a. (i) Show that $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

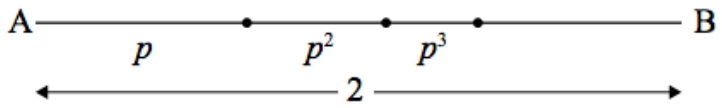
   (ii) Find a vector equation for ***L***.

   The line ***L*** also passes through the point $C(3, 1, p)$.

   b. Find the value of p.

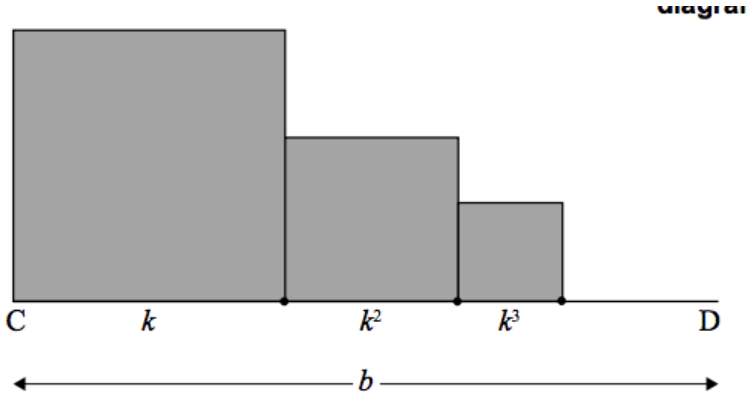
   c. The point D has coordinates $(q^2, 0, q)$. Given that $\overrightarrow{DC}$ is perpendicular to ***L***, find the possible values of q.

10. The following diagram shows [AB], with length 2 cm. The line segment is divided into an infinite number of smaller line segments. The diagram shows the first three segments. The length of the smaller line segments are p cm, p^2 cm, p^3 cm, …, where $0 < p < 1$.



   a. Show that $p = \frac{2}{3}$.

   The following diagram shows [CD], with length b cm, where $b > 1$. Squares with side lengths k cm, k^2 cm, k^3 cm, …, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.



   b. The total sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b.