Math SL PROBLEM SET 46

Section A (Short Answer)

1. **(C6.2 - N) (CI)** Find the first and second derivatives of the following functions:

(Cirrito 19.3, 618)

a. $f(x) = 3\sin(x) - 4\cos(x)$ b. $g(x) = 5\ln(x) - 2e^x$ c. $y = 6 - \ln\left(\frac{e^3}{x}\right)$

2. **(F2.2, 2.3, 2.5, 2.6 - R) (CI)** Let $f(x) = \ln(x)$ and let $g(x) = \frac{x-3}{2x}$, $x \neq 0$. Find:

(Cirrito 5.4, p148)

a. $g\ o\ f\ (e^3)$ b. $g^{-1}(4)$ c. The domain of $f\ o\ g\ (x)$

3. **(C6.1, C6.2 - N) (CI)** Find the equations of the lines that are normal to the following functions, at the given x value. **(Cirrito 20.2, p649)**

a. $y = \cos(x)$ at $x = \frac{5\pi}{6}$ b. $y = e^x + 1$ at $x = 1$ c. $y = 2 - \ln(x^2)$ at $x = 4$

4. **(V4.2 - R) (CA)** Vector equations for lines L_1 and L_2 are given below. **(Cirrito 12.6.1, p432)**

$$L_1 = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \quad \text{and} \quad L_2 = \begin{bmatrix} 5 \\ 2 \\ 7 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 7 \\ 5 \end{bmatrix}$$

a. Find the coordinates of the point where L_1 and L_2 intersect.
b. Find the angle between the two lines

5. **(T3.6 - R) (CA)** Consider the ΔABC, where AB =10 , BC = 7 and angle CAB = 32°.

(Cirrito 9.5.2, p297)

a. Find the two possible values of angle BCA .
b. Hence, find the two possible measures of AC.

6. **(P5.6 - R) (CI)** Two events, A and B, are such that $P(A) = \frac{9}{16}$ and $P(B) = \frac{3}{8}$ and $P(A \mid B) = \frac{1}{4}$. Find the probability that: **(Oxford 3.4, p85)**

a. Both events will happen.
b. Only one of the events will happen
c. Neither of the events will happen
d. Event A happens given that both events happen.

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Section B (Extended Response/Investigation)

7. **(V4.3 - E) (CA)** Lina is the captain of the oil tanker *Aristotle* and its path relative to the port of Orto (at the point (0,0)) is given by the vector equation, where t is time in hours after 12:00:

(Cirrito 12.7.2, p452)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 28 \end{bmatrix} + t \begin{bmatrix} 6 \\ -8 \end{bmatrix}$$

   a. Find the position of the *Aristotle* at 14:30.
   b. Find the velocity vector for the *Aristotle* and hence the speed of the oil tanker.
   c. Write the Cartesian equation for the path of the *Aristotle*.

A second ship, the *Euclid*, is piloted by Jana and is stationary with a position vector of $18\boldsymbol{i} + 4\boldsymbol{j}$.

   d. Show that the two ships will collide and find the time of the collision.

To avoid collision, the *Euclid* starts to move at 13:00 with a velocity vector of $5\boldsymbol{i} + 12\boldsymbol{j}$.

   e. Show that the position of the *Euclid* for $t > 1$ is given by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -8 \end{bmatrix} + t \begin{bmatrix} 5 \\ 12 \end{bmatrix}$$

   f. How far apart are the ships at 16:00?
   g. What is the bearing between the two ships at 16:00?

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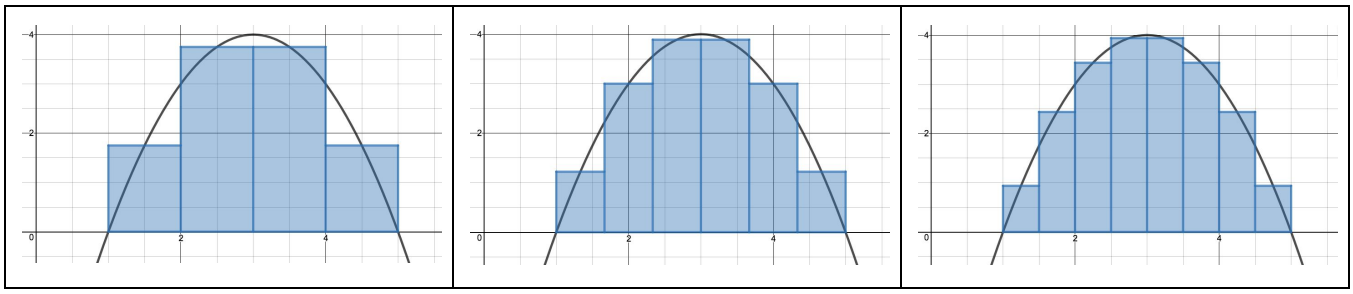
8. **(CA6.5 - N) (CA)** Omar knows the speed of an object is given by $v(t) = -t^2 + 6t - 5$ and he also knows that the area under a speed-time graphs gives the distance travelled by the moving object.

(Cirrito 22.5, p748)

a. Find the acceleration of the object at $t = 4$ seconds.

b. To estimate the distance travelled between $t = 1$ and $t = 5$, Omar has prepared three graphs, wherein he has drawn a number of rectangles under the curve (NOTE: the rectangles are "midpoint centered")



c. In each diagram,

i. find the height and width of each rectangle
ii. and then find the sum of the areas of the rectangles
iii. and hence, estimate the total area under the curve

d. How would you make your estimate more accurate?

e. Now, use your TI-84, graph the function and then from the graph, go to 2nd CALC and then move to item 7: $\int f(x)dx$. Input the lower limit to be $x = 1$ and then input the upper limit to be $x = 5$. What is the exact area? What does it actually represent in the context of this question?

f. In math notations, what you have just calculated is $\int_1^5 (-t^2 + 6t - 5)dt$