

# Math SL PROBLEM SET 37

## Section A (Short Answer)

1. **(V4.2 - N) (CA)** The lines  $L_1$  and  $L_2$  have the following equations: **(Cirrito 12.6.1, p432)**

$$L_1: x = 1 + 4t, y = 5 - 4t, z = -1 + 5t$$

$$L_2: x = 2 + 8s, y = 4 - 3s, z = 5 + s$$

- Is the point  $(-7, 13, -16)$  on  $L_1$ ?
  - What is meant by the term “skew lines”?
  - Show that these lines are skew.
2. **(A1.1 - E) (CA)** An arithmetic series has a first term of  $-4$  and a common difference of  $1$ . A geometric series has a first term of  $8$  and a common ratio of  $0.5$ . After how many terms does the sum of the arithmetic series exceed the sum of the geometric series? **(Cirrito 8.2.3, p261)**
3. **(A1.2 - N) (CI)** Mr. S. would like to solve the equation  $\log_4(x + 1) + \log_{\frac{1}{16}}(x + 1) = 1$ . **(Cirrito 7.4, p244)**
- Explain why he cannot start by using the addition rule of logarithms.
  - Re-express  $\log_{\frac{1}{16}}(x + 1)$  in terms of log base  $4$  (i.e.  $\log_4(??)$ )
  - Hence or otherwise, solve the equation  $\log_4(x + 1) + \log_{\frac{1}{16}}(x + 1) = 1$ .
4. **(C6.1, C6.3 - N) (CA)** Here is an equation of a cubic function,  $g(x) = x^3 - 2x^2 - 3x - 5$ .
- Provide a sketch of the function.
  - On what interval are the function values increasing?
  - On what interval is the function concave up?
  - What is the slope of this cubic function at  $x = 4$ ? Explain how you determined this value.
  - At what point(s) would you expect the slope of the tangent line(s) to be zero? Explain your reasoning.
  - On what domain does the function have negative slopes? Explain your reasoning.
5. **(A1.3 - N) (CA)** Consider the expression  $\left(\frac{2}{x^2} - x\right)^{15}$ , **(Cirrito 4.1.2, p100)**
- Find the first three terms of this expansion.
  - Find the last three terms of this expansion.
  - Find the constant term of this expansion OR justify that it does not exist.

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6. **(T3.5 - E) (CI)** Solve the following trigonometric equations on the domain of  $-90^\circ < x < 360^\circ$ . (NOTE: you may need a calculator to work out some of the inverse trig “stuff”) **(Cirrito 10.4, p351)**

- $2\sin(2x) = 3\cos(x)$
- $2\cos^2(x) + \sin(x) = -1$

## Section B (Extended Response/Investigation)

7. **(SP5.7 - N) (CA)** Given the following table for a discrete random variable,  $X$ : **(Cirrito C16.1, p533)**

$x$	1	2	3
$P(X=x)$	0.2	$1 - k$	

- Complete the table for the probability distribution. Express answer(s) in terms of  $k$ .
  - What range of values can  $k$  take? Give your answer in the form of  $a \leq k \leq b$ , where  $a, b \in \mathcal{Q}$ .
  - Find, in terms of  $k$ , the mean of the distribution.
  - Now suppose that  $k = 0.35$ . Find the mean and variance of the distribution.
  - Now suppose that  $k = 0.35$ . Determine  $P(X = 3 | X \geq 2)$
8. **(V4.2 - N) (CA)** Answer the following questions, involving the scalar (or dot) product: **(Cirrito 12.6, p440)**
- Find the measure of the angle between  $\mathbf{a}$  and  $\mathbf{b}$  if  $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ .
  - Find  $(\mathbf{a} + 3\mathbf{b}) \cdot (2\mathbf{a} - \mathbf{b})$  where  $\mathbf{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ .
  - Given that  $\mathbf{a} = 3\mathbf{i} - 5\mathbf{k}$  and given that  $\mathbf{b} = 2\mathbf{i} + 7\mathbf{j}$  and given that  $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ , find the vector  $\mathbf{d}$  such that  $\mathbf{a} \cdot \mathbf{d} = -9$  and  $\mathbf{b} \cdot \mathbf{d} = 11$  and  $\mathbf{c} \cdot \mathbf{d} = 6$ .