Math SL PROBLEM SET 33

Section A (Short Answer)

1. **(V4.3 - N)** **(CI)** Two lines have equations $r_1 = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix}$. **(Cirrito 12.7, Ex 20 on p449)**

   a. Show that the lines intersect.

   b. Find the coordinates of the point of intersection.

   c. Hence, determine the position vector of the intersection point and the magnitude of this position vector.

2. **(F2.1, F2.2, F2.4, F2.5, F2.6 - R)** **(CI)** Find the equation of the inverse functions of the following functions: **(Cirrito 5.4.2, p160)**

   a. $f(t) = 3e^{-0.25t} + 4$.

   b. $g(x) = \frac{2x-3}{x+4}$.

   c. $h(x) = 2x^2 + 4x - 6$

3. **(V4.1 - E)** **(CI)** Given that the vector $\boldsymbol{a}$ is defined as $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$ and that vector $\boldsymbol{b}$ is defined as $\boldsymbol{b} = 5\boldsymbol{i} - \boldsymbol{k}$, determine the following: **(Cirrito 12.3, p415)**

   a. The resultant vector $\boldsymbol{a} + \boldsymbol{b}$.

   b. The magnitude of $\boldsymbol{b}$ and then the magnitude of $4\boldsymbol{b}$.

   c. The magnitude of $\boldsymbol{a}$ and then the magnitude of $-3\boldsymbol{a}$.

   d. The resultant vector $5\boldsymbol{a} - 3\boldsymbol{b}$ and its magnitude.

   e. Explain what happens to a vector when you multiply it by a scalar.

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4. **(T3.6 - E) (CA)** For the ΔHKJ, side HK = 18 cm and side JK = 15 cm and ∠ JHK = 53°. Determine **(Cirrito 9.5.2, p297)**

   a. the measure of side HJ,
   b. and hence the area of the triangle.

5. **(T3.5 - E) (CI)** Solve the equation $2\sin(x) = \cos(2x)$ on the domain of $-2\pi \leq x \leq \pi$. **(Cirrito 10.2.2, p332)**

6. **(F2.7, C6.1 - E) (CI)** Find the value of a such that the line $y = 2x + a$ has exactly one intersection point with the parabola $y = x^2 + 3x + 2$. Hence, find the point at which the line and parabola intersect and hence, state the slope of the parabola at that point. **(Cirrito 2.4.4, p56)**

Section B (Extended Response/Investigation)

7. **(V4.3 - N) (CA)** A boat is initially located at a location defined by $\begin{pmatrix} 5 \\ -10 \end{pmatrix}$ and its motion along a straight line path is defined by $r_1 = \begin{pmatrix} 5 \\ -10 \end{pmatrix} + t\begin{pmatrix} -10 \\ 24 \end{pmatrix}$, where t is time in hours and distances are measured in kilometers. Determine **(Cirrito 12.7.2, p452)**

   a. The position of the boat after 1 hour.
   b. The speed of the boat.
   c. Find the direction that the boat is taking (its bearing) and hence, state its velocity.
   d. How long does it take the boat to reach the position defined by $(x, y) = -65\boldsymbol{i} + 158\boldsymbol{j}$?
   e. Does the boat ever reach the position defined by $\begin{pmatrix} -100 \\ 240 \end{pmatrix}$?
   f. How long does it take for the boat to travel 100km? Where is it located at that moment?

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8. **(V4.2 - N) (CA)** You know how to add vectors and subtract vectors and multiply vectors by scalar multiples, but how do you multiply vectors? Here are three video links to something called the dot product (or scalar product) of vectors. Videos 1 (https://www.youtube.com/watch?v=X5DifJW0zek) from Patrickjmt and Video #2 (https://www.youtube.com/watch?v=hkZtGslMcbQ) from mathispower4u will simply show you HOW to do the process but VIDEO #3 (https://www.youtube.com/watch?v=FrDAU2N0FEg) from BetterExplained will explain the WHY behind the HOW and give you a better intuitive understanding of dot product. After watching the videos, answer the following questions: **(Cirrito 12.6.1, p432)**

   a. Find the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ if $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{b} = 4\boldsymbol{i} + 2\boldsymbol{j} + 6\boldsymbol{k}$.

   b. Find the angle between the vectors $\boldsymbol{c}$ and $\boldsymbol{d}$, if $\boldsymbol{c}$ is parametrically defined as $x(t) = 2 - 5t$ and $y(t) = -3 + 3t$ and $\boldsymbol{d}$ is defined as $\frac{2-3x}{5} = \frac{5x+8}{3}$.

   c. Are the vectors $u = \begin{pmatrix} -8 \\ 2 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} -2 \\ 5 \\ -13 \end{pmatrix}$ parallel, perpendicular or neither?