Math SL PROBLEM SET 31

Section A (Short Answer)

1. (**A1.1 - E**) (**CA**) For the following geometric sequences, A_n, determine u_{10}, S_{10} as well as S_∞: **(Cirrito 8.2.4, p263)**

   a. In $\{A_n\}$ where the terms are 200, 150, 112.5, 84.375, …..
   b. In $\{A_n\}$ where $u_5 = 24$ and $u_8 = \frac{24}{27}$.
   c. In $\{A_n\}$ where the terms are 100, 110, 121, 133.1, …….

2. (**F2.6, F2.8 - R**) (**CA**) Which is best: (i) to have money in a bank that pays 9 percent annual interest, (ii) one that pays 9/12 percent monthly interest, (iii) or one that pays 9/365 percent daily interest? Show calculations to support your decision. (NOTE: a bank is said to compound its annual interest when it applies a fraction of its annual interest to a fraction of a year.) **(Cirrito 7.1.5, p207)**

3. (**F2.6, F2.8 - R**) (**CA**) (CONTINUATION) Inflation in the country of Mathylvania has reached alarming levels. Many banks are paying 100 percent annual interest, some banks are paying 100/12 percent monthly interest, a few are paying 100/365 percent daily interest and so forth. In trying to make sense of all these different bank promotions, Daniel decides to graph the function $E(x) = \left(1 + \frac{1}{x}\right)^x$. What does the graph reveal about the sequence $v_n = E(n) = (1 + \frac{1}{n})^n$, where n is a positive integer? Calculate these specific values: v_1, v_{12}, v_{365}, $v_{3156000}$. **(Cirrito 7.1.5, p207)**

4. (**F2.6, F2.8 - R**) (**CA**) (CONTINUATION) The sequence in the previous problem has a limiting value. This sequence is so important that a special letter is reserved for the limiting value (as is done for π). We write $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$. The $\lim_{n\to\infty}$ means "as n approaches ∞" or "as n gets very large". For some additional work with this sequence, use your calculator to evaluate $\lim_{n\to\infty}(1 + \frac{0.09}{n})^n$. Make up a story to go with the question. **(Cirrito 7.1.5, p207)**

5. (**C6.1 - N**) (**CA**) Determine the value of the following "limits" ⇒ i.e. determine the limiting value of $f(x)$ as per $\lim_{n\to\infty} f(x)$, where $f(x)$ is:

   a. Let $f(x) = \frac{2x-1}{x+3}$, so in other words, evaluate $\lim_{n\to\infty} \frac{2x-1}{x+3}$.
   b. Let $f(x) = 20\left(\frac{3}{4}\right)^x$, so in other words, evaluate $\lim_{n\to\infty} 20\left(\frac{3}{4}\right)^x$.
   c. Let $f(x) = 2x^3 - x$, so in other words, evaluate $\lim_{n\to\infty} 2x^3 - x$.
   d. Let $f(x) = \tan^{-1}(x)$, so in other words, evaluate $\lim_{n\to\infty} \tan^{-1}(x)$

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6. (**C6.1 - N**) (**CA**) Continuing this work with understanding limits, evaluate the following limits (in other words, determine the limiting function value of $f(x)$ in the following scenarios ……)

1) $\lim_{x\to -1} 5$

2) $\lim_{x\to -\frac{5}{2}} (-x+2)$

3) $\lim_{x\to 2} (x^3 - x^2 - 4)$

4) $\lim_{x\to 1} \left(-\frac{x^2}{2} + 2x + 4\right)$

5) $\lim_{x\to 3} -\sqrt{x+3}$

6) $\lim_{x\to \frac{3}{2}} -\sqrt{2x+4}$

Section B (Extended Response/Investigation)

7. (**V4.3 - N**) (**CI**) The line L is defined by the parametric equations $x(k) = 4 - 5k$ and $y(k) = -2 + 3k$. **(Cirrito 12.7.1, p444)**

   a. Find the coordinates of three points on L.
   b. Find the value of k that corresponds to the point (14, -8)
   c. Show that the point (-1,4) does not lie on the line L.
   d. Find the vector and Cartesian forms on the line L.
   e. A second line, M, is defined parametrically by $x(t) = a + 10t$ and $y(t) = b - 6t$. Describe the relationship between M and L if $a = 4$ and $b = -2$.
   f. Find the point at which the line L intersects with the line N, if the line N is defined parametrically as $x(\lambda) = 5 - 4\lambda$ and $y(\lambda) = -3 + 2\lambda$.

8. (**PS5.8 - N**) (**CA**) To continue working with the concept of **binomial probability distributions,** answer the following questions that involve the following scenario: You are given 5 "unfair" dice, in which the probability of rolling a 5 or 6 is only 25% (or $\frac{1}{4}$) and thus the probability of NOT rolling a 5 or 6 is 75% (or likewise $\frac{3}{4}$). Our "experiment" consists of rolling each of the 5 dice once and looking to see how many 5s or 6s we have at the end of the "experiment". **(Cirrito 16.3.4, p544)**

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a. Use a tree diagram to show ALL possible outcomes of this experiment.

b. How many different ways can you get 5s or 6s appearing 3 times?

c. Determine how probable it is that we get the 5s or 6s appearing:
   i. Once
   ii. Twice
   iii. Three times
   iv. Four times
   v. Five times

d. Draw a probability histogram, where the x-axis is the number of 5s or 6s appearing and the y-axis is the probability of getting that number of 5s or 6s..

e. Expand $(p + q)^5$.

f. Now we will let p = probability of rolling a 5 or 6 (so $p = \frac{1}{4}$) and if we let q = probability of NOT rolling a 5 or 6 (so $q = \frac{3}{4}$). Use your expansion from Q(e) to calculate the probability of getting 5s or 6s to appear twice from our "experiment".

g. Now let's use our calculator to do the same calculation in one step!!! Go to 2nd VARS (to the DISTR menu) and from there, scroll down to A: binompdf(. You will need to imput the number of trials (5) as well as the value of p (¼) as well as the x value (2) and then "paste" that to the home screen. Finally, hit enter …….