

Math SL PROBLEM SET 20

Section A (Short Answer)

1. **(T3.1 - N) (CA)** The diagram shows an equilateral triangle ABC with sides of length 6 cm.

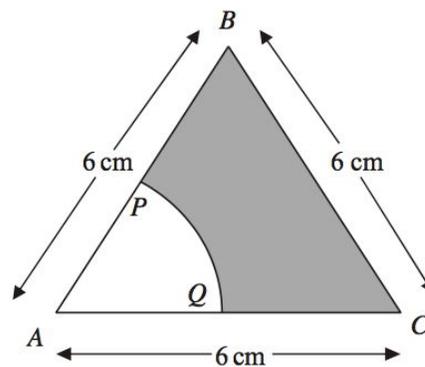
(Cirrito 9.4, p287; Cirrito 9.7, p309)

P is the midpoint of AB.

Q is the midpoint of AC.

APQ is a sector of a circle, centre A.

- Calculate the length of the arc PQ of the sector
- Calculate the area of the shaded region. Give your answer correct to 3 significant figures.



2. **(T3.5 - R) (CI)** The population (in thousands) of a species of butterfly in a nature sanctuary is modelled by the function:

$$P(t) = 3 + 2 \sin\left(\frac{3\pi t}{8}\right), \quad 0 \leq t \leq 12$$

where t is the time in weeks after scientists first started making populations estimates.

(Cirrito 10.5, p361)

- What is the initial population?
 - What are the largest and smallest populations?
 - When does the population exceed 4,000 butterflies?
3. **(T3.5 - E) (CI)** Draw the two special right triangles as well as graphs of $y = \sin(x)$ and $y = \cos(x)$. Label the maximums, minimums and intercepts of these two graphs. *(Cirrito 10.4, p351)*

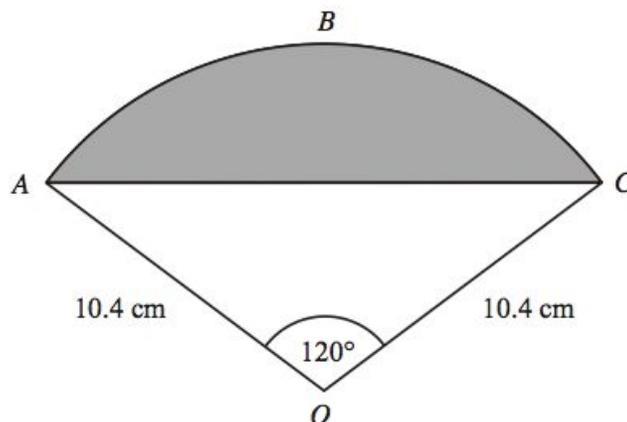
- Solve $\sqrt{2} \cos(x) + 1 = 0$ for $-360^\circ \leq x \leq 360^\circ$
- Solve $\sin^2(\theta) - 1 = 0$ for $0 \leq \theta \leq 4\pi$

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4. **(T3.1 - N) (CA)** The diagram shows a sector OABC of a circle with centre O.

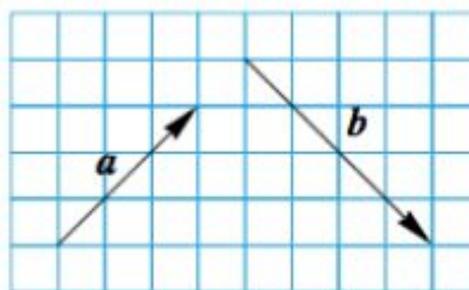
Given that $OA = OC = 10.4$ cm and angle $AOC = 120^\circ$. (*Cirrito 9.4, p287; Cirrito 9.7, p309*)

- a. Calculate the length of the arc ABC of the sector. Give your answer correct to 3 significant figures.
- b. Calculate the area of the shaded segment ABC. Give your answer correct to 3 significant figures.



5. **(V4.1 - N) (CI)** Using the vectors shown in the diagram, draw the vectors (*Cirrito 12.3, p415*)

- a. $a - b$
- b. $b - 2a$
- c. $2b - 3a$
- d. $\frac{1}{2}(b + 2a)$



6. **(A1.1 - E) (CI)** Find the value of p so that $p + 5$, $4p + 3$ and $8p - 2$ are three successive terms of an arithmetic sequence. (*Cirrito, 8.1, p241*)
7. **(A1.1 - E) (CI)** Three successive terms of a geometric sequence are $2k + 2$, $5k + 1$ and $10k + 2$. Find the value(s) of k . (*Cirrito, 8.2, p252*)

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Section B (Extended Response/Investigation)

8. (**T3.2, T3.3 - N**) (CI) Determine the sine and cosine ratios of $\frac{\pi}{3}$, $\frac{3\pi}{4}$ and $-\frac{5\pi}{6}$. Use these ratios to determine the value of: (*Cirrito 10.1.2, p316; Cirrito 10.2.1, p327; Cirrito 10.2.2, p332*)
- the expression $\sin^2(x) + \cos^2(x)$ for $x = \frac{\pi}{3}$ and $x = \frac{3\pi}{4}$ and $x = -\frac{5\pi}{6}$.
 - the expression $\frac{\sin(x)}{\cos(x)}$ for $x = \frac{\pi}{3}$, $\frac{3\pi}{4}$ and $-\frac{5\pi}{6}$ and compare to the value of $\tan(x)$ for the same angles of $x = \frac{\pi}{3}$, $\frac{3\pi}{4}$, $-\frac{5\pi}{6}$
 - the expression $\cos^2(x) - \sin^2(x)$ for $x = \frac{\pi}{3}$ and $x = \frac{3\pi}{4}$ and $x = -\frac{5\pi}{6}$ and compare it to the value of $\cos(2x)$ for $x = \frac{\pi}{3}$, $\frac{3\pi}{4}$, $-\frac{5\pi}{6}$
9. (**A1.1 - E**) (CA) Here are two more geometric series: (*Cirrito 8.2.4, p263*)
- $\frac{9}{2} + 3 + 2 + \frac{4}{3} + \dots$
 - $240 - 60 + 15 - 3.75 + \dots$
- b. For each series,
- Find the common ratio, r .
 - Use your calculator to find S_{10} , S_{15} and S_{20} . Record the complete value (no rounding)
- c. Do you notice any patterns? Why do you think this is happening?
- d. Now use your calculator to evaluate S_{50} . Do you think your calculator is correct? Why or why not?