Math SL PROBLEM SET 13

Section A (Short Answer)

1. (**SP5.6, SP5.7 - R,N**) (**CI**) As a result of a certain random experiment, the events A and/or B may occur. These events are independent and $P(A) = 0.5$ and $P(B) = 0.2$. *(Cirrito 15.2, p508; Oxford 3.2, p68)*

    a. Find the probability that both A and B occur.

    b. Find the probability that neither A nor B occurs.

    c. Find the probability that either A or B occurs.

    d. Find the probability that A happens, given that B has occurred. What do you notice?

    e. Find the probability that B happens, given that A has occurred. What do you notice?

    f. Let X denote the random variable which counts how many of the two events occur at a given time. Thus, for example, $X = 0$ if neither A nor B occur

       i. What would $X = 1$ mean?

       ii. Find $P(X = x)$ for $x = 0, 1, 2$.

2. (**A1.2, F2.6 - R**) (**CA**) The number of bacteria in a culture is modeled by the function $n(t) = 10\, e^{0.22\, t}$, where t is time in hours and $n(t)$ is the number of bacteria. *(Cirrito 7.2, p209)*

    a. What is the hourly rate of growth of this bacterium population? Express your answer as a percentage.

    b. What is the initial population of the culture (at t = 0 hours)?

    c. Evaluate and interpret $n(15)$.

    d. Solve and interpret the equation $500 = n(t)$.

    e. What is the doubling time for this bacterial population?

3. (**A1.2, F2.6 - E**) (**CI**) Consider the system of equations $(5^x)(25^{2y}) = 1$ and $(3^{5x})(9^y) = \frac{1}{9}$. *(Cirrito 7.1.2, p200)*

    a. Show that this system of equations **implies** that $x + 4y = 0$ and $5x + 2y = 2$

    b. Hence, solve the system of equations.

4. (**SP5.1, SP5.3 - R**) (**CI**) Here is a frequency distribution table, showing the number of hours a typical SL Math student spends per night on Math homework. Use the data in this table to: *(Cirrito 13.2, p471)*

x	0	1	2	3	4	5
frequency	1	3	6	6	7	1

   a. Construct a frequency histogram and hence a frequency polygon.

   b. Construct a cumulative frequency graph.

   c. Calculate the 3 measures of central tendency.

   d. Construct a box and whisker plot

5. (**SP5.2 - N**) (**CA**) Find the standard deviation for the following test scores. Use the chart below to record the intermediate steps/calculations. Confirm your final answer using the TI-84. The test scores are: 85, 100, 92, 96, 87, 94   *(Cirrito 13.4.2, p478)*

| Score	Difference from the Mean	(Difference from the Mean)^2
	Sum of (Difference from the Mean)^2	

A. Mean:_____   **B. n:**_____

C. Sum of (Difference from the Mean)^2 divided by (n):_____

D. Standard deviation $\left(\sqrt{\dfrac{diff.fromMean)^2}{n}} \right)$ is_____

(Teaching NOTE: Here are two graphs that show what standard deviation (or variance in this case) mean as far as a picture of a statistical distribution is concerned ⇒ note the different "spread or variation" of the data that is implied in the visuals)
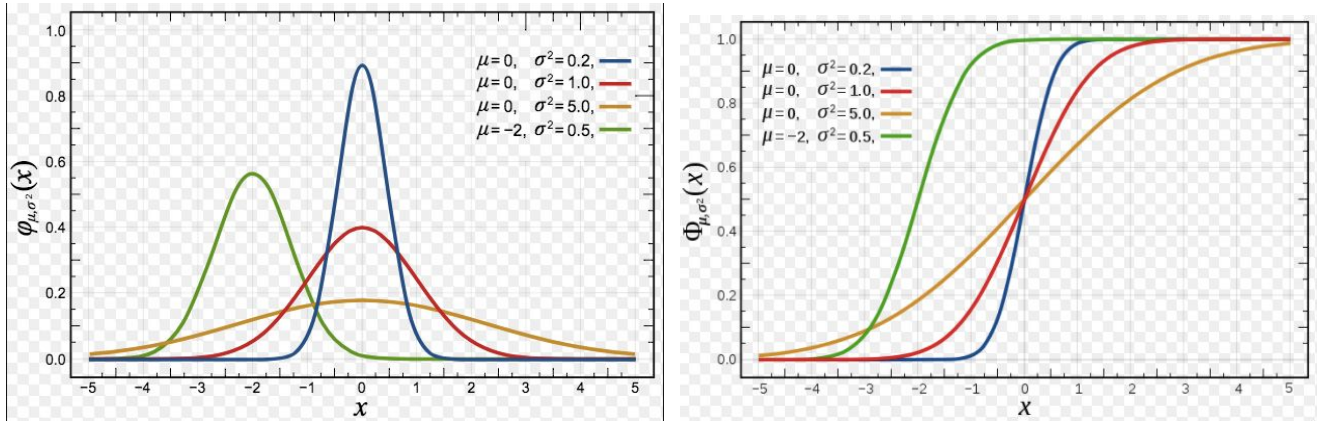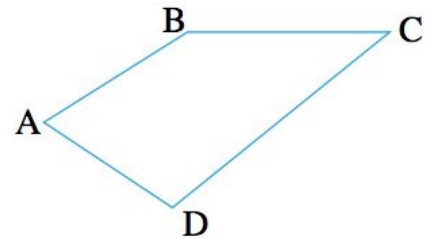

6. (**V4.1 - N**) (**CI**) *Geometric Vectors* are vectors not related to any coordinate system. For example, the directed line segment $\overrightarrow{AB}$ (see picture) where A is called the initial (start, tail) point and B is called the final (end, terminal, head or tip) point. **(*Cirrito 12.2, p410*)**
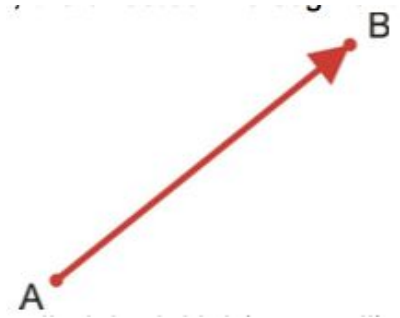
For the shape shown, find a single vector which is equal to
(a)   **AB + BC**
(b)   **AD + DB**
(c)   **AC + CD**
(d)   **BC + CD + DA**
(e)   **CD + DA + AB + BC**

7. (**A1.2 - E**) (**CA**) Pattern Set: Use your calculator to evaluate each of the following expressions in the each row and then make a summary statement *(Cirrito 7.4, p221)*

$\ln\left(e^2\right)$	$\ln\left(e^3\right)$	$\ln\left(e^4\right)$	$\ln\left(e^0\right)$	$\ln\left(e^{-2}\right)$	$\ln\left(\dfrac{1}{e^4}\right)$	$\ln\left(\sqrt{e}\right)$	In general?
$\ln\left(2^2\right)$	$\ln\left(2^3\right)$	$\ln\left(2^4\right)$	$\ln\left(2^0\right)$	$\ln\left(2^{-1}\right)$	$\ln\left(\dfrac{1}{2^4}\right)$	$\ln\left(\sqrt{2}\right)$	In general?
$\log_3 2$	$\log_3 4$	$\log_3 8$	$\log_3 16$	$\log_3 32$	$\log_3 0.5$	$\log_3 0.25$	In general?

Section B (Extended Response/Investigation)

8. (**F2.2, F2.7, F2.8 - E**) (**CA**) Rumours of an imminent take-over by a large electronics company has forced the value of shares of Smith Electronics to rise. Unfortunately, one week later, Smith Electronics declared that the take-over would **not** happen. Consequently, the value of the shares of Smith Electronics now has changed and their value is now modelled by the equation below, where t is time in weeks since the rumour started and $V(t)$ is value in cents.

$$V(t) = \frac{400}{t^2 - 2t + 2}$$

   a. Sketch the graph of the function V.

   b. What was the value of the shares in Smith Electronics before the rumour started?

   c. What is the maximum value that Smith Electronics reaches?

   d. What is the average rate of change of the value of shares between week 3 and week 5?

   e. Mr Dunham bought shares in Smith Electronics before the rumour started. If he is prepared to sell them at 50% profit, when should he sell his shares?

9. (**T3.3 - N**) (**CA**) **_Identities_**: An algebraic identity is an algebraic equation that true for every value of x. For example, the equation $(x + 2)^2 = x^2 + 4x + 4$ is going to be true, regardless of what number you substitute in for x.   **_(Cirrito 10.2, p327)_**

    a. Substitute in $x = 1$, $x = 2$, $x = 5$ into BOTH sides of the equation and see what happens.

    b. Is the algebraic equation $x^2 + y^2 = (x + y)^2 - 2xy$ an identity? True or False? Prove it.

    c. We also have trigonometric identities. Given the equation $\sin^2(x) + \cos^2(x) = 1$, use $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$ to show that $\sin^2(x) + \cos^2(x) = 1$ could be an identity. How would you prove it?

    d. Given the expression $2\sin(x)\cos(x)$:

        i.   Evaluate $2\sin(x)\cos(x)$ for $x = 30°$. Then, use your answer to evaluate $\sin^{-1}(\text{ANS})$.

        ii.   Evaluate $2\sin(x)\cos(x)$ for $x = 45°$. Then, use your answer to evaluate $\sin^{-1}(\text{ANS})$.

        iii.   What observation do you make?