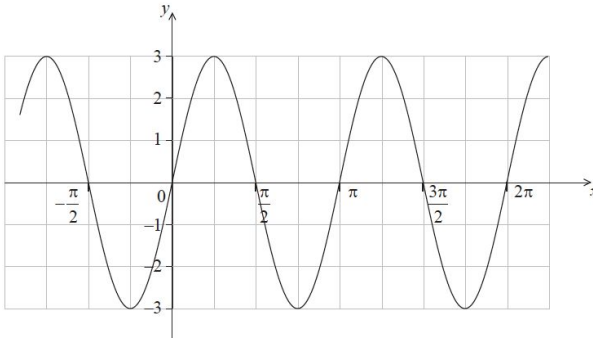
Math SL PROBLEM SET 101

Section A (Short Answer)

1. (CI) A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

   a. Write down the probability that the first marble drawn is red.
   b. Complete a tree diagram.
   c. Find the probability that both marbles are blue.

2. (CI) Let $f(x) = a \sin bx$, where $b > 0$. The following diagram shows part of the graph of f.



   a. (i) Find the period of f.
      (ii) Write down the amplitude of f.
   b. (i) Write down the value of a.
      (ii) Find the value of b.

3. (CI) Let $g(x) = \frac{1}{x\ ln(x)}$. (a) Find $g'(x)$. (b) Find $\int g(x)dx$.

4. (CI) Let $f(x) = e^{-2x}$. (a) Write down $f'(x)$, $f''(x)$, and $f^{(3)}(x)$. (b) Find an expression for $f^{(n)}(x)$.

5. (CI) The following cumulative frequency diagram shows the lengths of 160 fish, in cm. (Diagram on LAST PAGE)

   a. Find the median length.

   The following frequency table also gives the lengths of the 160 fish.

   b. (i) Write down the value of p.
      (ii) Find the value of q.

Length x cm	$0 \le x \le 2$	$2 < x \le 3$	$3 < x \le 4.5$	$4.5 < x \le 6$
Frequency	p	50	q	20

6. (CI) Let $f(x) = ax^3 + bx$. At $x = 0$, the gradient of the curve of f is 3. Given that $f^{-1}(7) = 1$, find the value of a and of b.

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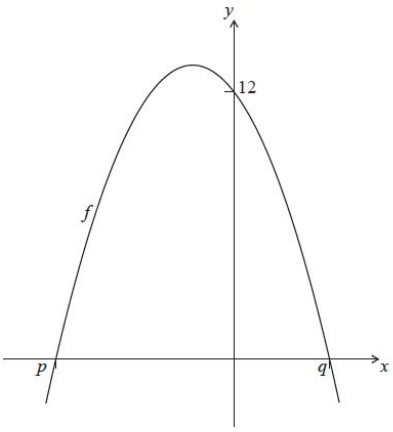
7. A bag contains black and white chips. Rose pays \$10 to play a game where she draws a chip from the bag. It is given that P(black) = 0.4 and that P(white) = 0.6. Rose gets no money if she draws a white chip, and gets \$$k$ if she draws a black chip. The game is fair. Find the value of k .

Section B (Extended Response)

8. (CI) Let $f(x) = a(x + 3)(x - 1)$. The following diagram shows part of the graph of f . The graph has x-intercepts at $(p, 0)$ and $(q, 0)$, and a y-intercept at $(0, 12)$.



   a. (i) Write down the value of p and of q .
      (ii) Find the value of a .
   b. Find the equation of the axis of symmetry of the graph of f .
   c. Find the largest value of f .

   The function f can also be written as $f(x) = a(x - h)^2 + k$.

   d. Find the value of h and of k .

9. Let P and Q have coordinates (1,0,2) and (–11,8,m) respectively.

   a. Express PQ in terms of m.
   b. Let a and b be perpendicular vectors, where $a = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}n$ and $b = -3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$. Find n .

   Given that PQ is parallel to b ,
   c. (i) express PQ in terms of b ;
      (ii) hence find m .

   In part (d), distance is in metres, time is in seconds.
   d. A particle moves along a straight line through Q so that its position is given by $r = c + ta$.
      (i) Write down a possible vector c .
      (ii) Find the speed of the particle.

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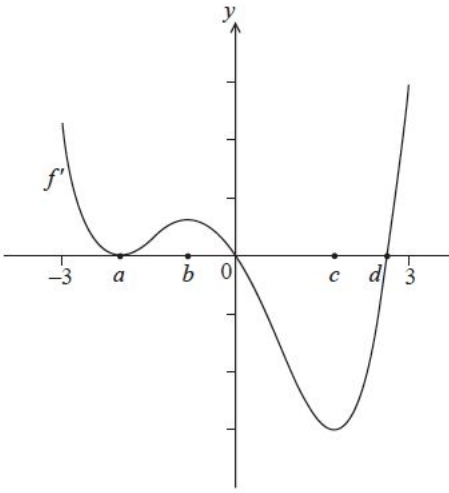
10. (CI) Consider a function f with domain $-3 < x < 3$. The diagram shows the graph of f', the **derivative** of f. The graph of f' has x-intercepts at $x = a$, $x = 0$, and $x = d$. There is a local maximum at $x = b$ and local minima at $x = a$ and at $x = c$.



a. Find all possible values of x where the graph of f is decreasing.

b. Find the value of x where the graph of f has a local minimum and justify your answer.

The total area of the region enclosed by the graph of f' and the x-axis is 15.

c. Given that $f(a) = 3$ and $f(d) = -1$, find the value of $f(0)$.