

Math SL EXPLORATION LAB 2

Applications of Sequences and Series

PART A - The BASICS

1. For the arithmetic sequence 2, 8, 14, 20,, determine:
 - a. The 50th term
 - b. The sum of the first 40 terms
 - c. The number of the term whose value does NOT exceed 500,000
 - d. Are there any “special formulas” that you may want to remember for working with arithmetic sequences and series?

2. For the geometric sequence 6, 12, 24, 48, 96,, determine
 - a. The 10th term
 - b. The sum of the first 14 terms
 - c. The value for n such that $\sum_{i=1}^n 6(2)^{n-1} < 50,000$
 - d. Are there any special “formulas” that you may want to remember for working with geometric sequences and series?

3. Your grandparents put aside \$100 for you on your first birthday. Every following year, they put away \$125 more than they did in the previous year. How much money will have been put aside by the time you are 21?

4. A catering company has 15 customer orders for its first month of operation. For each subsequent month afterward, the company has double the number of orders than the previous month.
 - a. How many orders in total did the company have at the end of its second year?
 - b. The company will hire a new employee once the total number of orders first exceeds 15,000. In what month (and year) will this occur?

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PART B - The PROBLEMS

5. Problem 1 - Allergy Medicine

It is estimated that 1 in every 7 Canadians suffers from seasonal allergies such as hay fever. A typical treatment for hay fever is anti-histamines. Tim decides to try BRAND X antihistamines. The label says:

- The half life of the antihistamine in the body is 16 hours
- For his size, the maximum relief is felt when there are 150 mg to 180 mg in the body. Side effects (sleepiness, headaches, nausea) can occur when more than 180 mg are in the body.
- Each pill contains 30 mg of active ingredient. It is unhealthy to ingest more than 180 mg within a 24 hour period

PROBLEM: How many hay fever pills should Tim take and how often should he take them? Work through the following 4 questions to help develop an answer to this problem.

- A. What are some conditions that would be reasonable when taking medications? (For example, think about dosages as well as times of the day when you take the medication)
- B. Determine three different schedules for taking the pills, considering the appropriate amounts of the medication to ingest and your conditions in PART A.
- C. For each of your schedules, determine the amount of the medication present in Tim's body for the first few days.
- D. Based upon your calculations in PART C, which schedule is best for Tim? Is another schedule more appropriate?

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6. Problem 2 - Saving for Retirement

Steve, Carol and Lisa get their first full-time jobs and talk about saving money for their retirement. They are each 22 years old and plan on working until they are 55.

Steve starts investing immediately and puts aside \$150 per month. Carol wants to enjoy life a bit and decides to start contributing when she turns 30. Lisa thinks that both her friends are starting too early and decides to wait until she is 42 before starting to save.

Assuming that Steve, Carol and Lisa are each earning 9% p.a. compounded monthly. Carol and Lisa want to accumulate the same amount as Steve upon their retirement. When they retire, Steve wants his investment to last 10 years, Carol wants hers to last 15 years and Lisa wants hers to last 20 years.

PROBLEM: How much will Steve, Carol and Lisa be able to withdraw monthly upon retirement? Work through the following 5 questions to help develop an answer to this problem.

- A. What strategies will you use to solve this problem? Justify your strategies.
- B. How much money will Steve have accumulated by the time he is 55? What assumptions are you making?
- C. How much will Lisa and Carol have to deposit each month to meet their goals?
- D. Whose investment plan is “the best”? Justify your answer.
- E. How much will each person be able to withdraw from their retirement fund (after the age of 55 of course) each month?
- F. **EXTENSION:** Let’s assume the same investment return (9% p.a. compounded monthly) over the life of your investment and let’s ignore the effect of inflation for now. You wish to retire and have monthly withdrawals from your retirement fund of \$4,000 per month. So outline a savings strategy that you can use to meet this condition. State any assumptions that you are making in designing your investment strategy.