BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

1. You are required to perform the following algebraic manipulations with polynomial functions: {6,8}

   (i) $f(x) = 2(x-2)^2(1-x)$ and (ii) $f(x) = -3x(2x-3)(x+3)$

   a. expand the polynomial

   b. detemine the leading coefficient, the degree, and the constant term

   c. State the leading term and predict the end behaviour of these cubic polynomials

   d. from FACTORED FORM find the y-intercept and $f(-2)$

   e. from STANDARD FORM find the y-intercept and $f(-2)$

   f. Sketch what you think the polynomial should look like

   g. Graph it using your TI-84 as well as DESMOS (but only after answering questions a – f).

2. (CI) The following questions will help you to continue learning how to factor polynomials. {2,10}

   a. Which binomials are factors of $P(x) = 2x^3 - x^2 - 7x + 6$?

      Either (i) $x + 3$ OR (ii) $2x - 3$ OR (iii) neither OR (iv) both

   b. Which binomials are factors of $P(x) = -2x^4 - 7x^3 + 22x^2 + 63x - 36$?

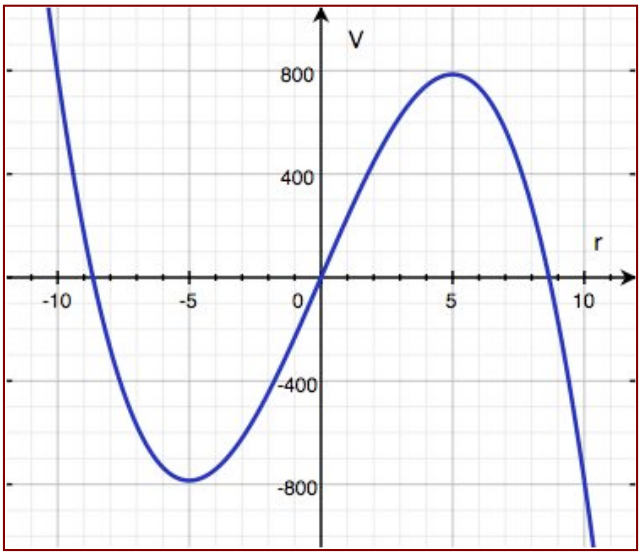
      Either (i) $x - 1$ OR (ii) $x - 3$ OR (iii) neither OR (iv) both

   c. Given the polynomial $P(x) = x^3 - 2x^2 - 21x - 18$, is $x = 6$ a zero of $P(x)$? is $x = -2$ a zero of $P(x)$?

   d. Given the polynomial $P(x) = x^4 - 3x^3 + 3x^2 - 3x + 2$, is $x = 2$ a root of $P(x)$? is $x = -2$ a root of $P(x)$?

3. Jeannie wishes to construct a cylinder closed at both ends. The given graph of a cubic polynomial function, V, used to model the volume of the cylinder as a function of the radius if the cylinder is constructed using 150π cm^3 of material. Use the graph to answer the questions below. Estimate values to the nearest half unit on the horizontal axis and to the nearest 50 units on the vertical axis. {8}

   a. What are the zeros of the function V?
   b. What is the relative maximum value of V, and where does it occur?
   c. The equation of this function is $V(r) = c(r^3 - 72.25r)$ for some real number c. Find the value of c so that this formula fits the graph.
   d. Use the graph to estimate the volume of the cylinder with $r = 2$ cm.
   e. Use your formula for V to find the volume of the cylinder when $r = 2$ cm. How close is the value from the formula to the value on the graph?



4. The following question deal with one application of rational functions ➔ inverse variation. {8}

   a. The number of hours, h, it takes for a block of ice to melt varies inversely as the temperature, t. If it takes 2 hours for a square inch of ice to melt at 65º, find the temperature at which it takes 7.25 hours to melt.

   b. In kickboxing, it is found that the force, f, needed to break a board, varies inversely with the length, l, of the board. If it takes 5 lbs of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long?

   c. The number of miles per gallon of gasoline that a vehicle averages varies inversely as the average speed the car travels. A vehicle gets 13 miles per gallon at 52 mph. How many miles per gallon will it get at 64 mph?

5. (CA) The concentration, C (in mg/litre), of a drug in a patient's bloodstream t hours after injection is modeled by the rational function $C(t) = \frac{t}{2t^2+1}$. Use your TI-84 to help determine the following: {4,8}

   a. How much drug is in the patient's blood after 4 hours?

   b. At what time does the patient have the maximum amount of drug in their blood? What is this maximum amount?

   c. In what domain interval is the concentration of the drug increasing?

   d. Where does the function have any asymptotes? Why does this make sense?

   e. The patient requires a second injection of the drug when the drug concentration reaches 0.1 mg/liter. When should the patient receive the next injection?

   f. Sketch the function in your notes. What domain should you use?

6. In this question, you will practice converting from linear/linear form to transformation form.

   a. From the form $f(x) = \frac{ax+b}{cx+d}$, state the equations for the vertical and horizontal asymptotes.

   b. Convert the following functions from the form $f(x) = \frac{ax+b}{cx+d}$ to the form $f(x) = \frac{M}{cx+d} + N$

   i. $f(x) = \frac{9x-2}{3x+4}$ ii. $f(x) = \frac{8x+5}{2x-8}$ iii. $f(x) = \frac{3x-2}{2x+5}$

You can use long division ➔ see video for example ➔ https://youtu.be/H94ma2ofGuc

You can use synthetic division ➔ Example with synthetic division if you are curious ➔ Remember, if the coefficient of the x term in the divisor is not 1, you must divide the quotient by that value too!

$f(x) = \frac{10x-17}{5x-1}$

$\frac{1}{5}$ | 10 −17
2
10 | −15

$f(x) = \frac{10}{5} - \frac{15}{5x-1} \rightarrow f(x) = 2 - \frac{15}{5x-1}$

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

1. (CI for a challenge) For each of the following, determine (where possible):

   a. The co-ordinates of any holes in the graph
   b. The co-ordinates of the x- and y-intercepts
   c. The equation of the vertical asymptotes
   d. The type and equation of non-vertical asymptotes

   i. $y = \dfrac{1 - x^2}{x}$

   ii. $y = \dfrac{2x + 2}{x^2 - 3x - 4}$

   iii. $y = \dfrac{x^2 - x - 2}{x^2 - 1}$

   iv. $y = \dfrac{x^3 - 8}{x - 2}$

2. Determine the Partial Fraction Decomposition of each of the following expressions:

   a. $\dfrac{22 + 7x}{x^2 + 5x + 4}$

   b. $\dfrac{7x - 44}{4x^2 + 25x - 21}$

   c. $\dfrac{-x - 47}{x^2 - 11x + 24}$

   d. $\dfrac{5 - 38x}{8x^2 + 2x - 1}$