BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

1. With this question, you will (hopefully) see the connection between (i) evaluating a polynomial and (ii) synthetic division and (iii) determining factors. {6,10,11}

   a. Example #1

      i. Evaluate P(-2) if $P(x) = 2x^4 + 5x^3 - 10x^2 - 15x + 18$.
      ii. Divide $P(x) = 2x^4 + 5x^3 - 10x^2 - 15x + 18$ by $x + 2$ using synthetic division.
      iii. What is the remainder after the division? What is the quotient after the division?
      iv. Using your QUOTIENT (should be a cubic polynomial) evaluate $P(1)$.
      v. Is $x - 1$ a factor of the cubic quotient? Is $x - 1$ a factor of the original quartic?.
      vi. Hopefully, you have now determined two factors of $P(x) = 2x^4 + 5x^3 - 10x^2 - 15x + 18$. Determine the other two factors.
      vii. Sketch the polynomial.

   b. Example #2

      i. Evaluate P(1) if $P(x) = -4x^4 - 6x^3 + 28x^2 + 18x - 36$
      ii. Divide $P(x) = -4x^4 - 6x^3 + 28x^2 + 18x - 36$ by $x - 1$ using synthetic division.
      iii. What is the remainder after the division? What is the quotient polynomial after the division?
      iv. Evaluate $P(2)$ in your QUOTIENT polynomial
      v. Divide your QUOTIENT polynomial by $x - 2$ using synthetic division.
      vi. Hopefully, you have now determined two factors of $P(x) = -4x^4 - 6x^3 + 28x^2 + 18x - 36$. Determine the other two factors.
      vii. Sketch the polynomial

2. (CA) Given the functions $f(x) = \dfrac{2x-5}{x-4}$ and $g(x) = -x^3 + 5x^2 + 2x$, use your TI-84 to determine: $\{15\}$

   a. the solution to the equation $f(x) = g(x)$ (i.e. the intersection points)

   b. HENCE, detemine the domain intervals in which $f(x) > g(x)$

3. (CI) Given the polynomial $p(x) = (3-x)(x-1)(x+4)$; $\{1,2,6\}$

   a. Expand the polynomial and write the equation in standard form.

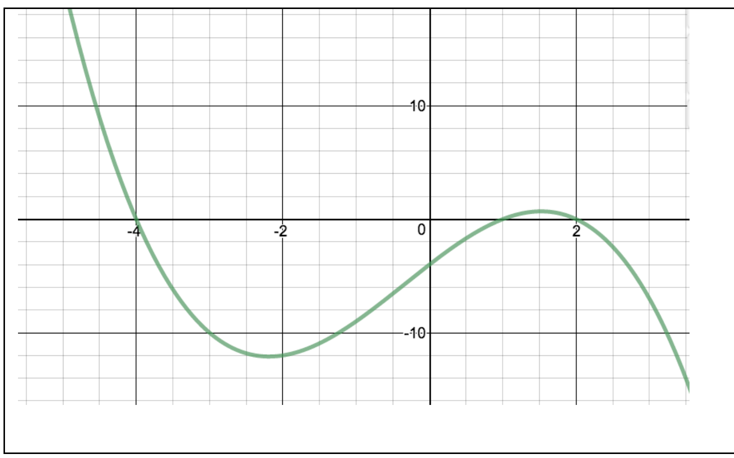
   b. Determine the value of the leading coefficient and hence, predict the end behaviour of this polynomial

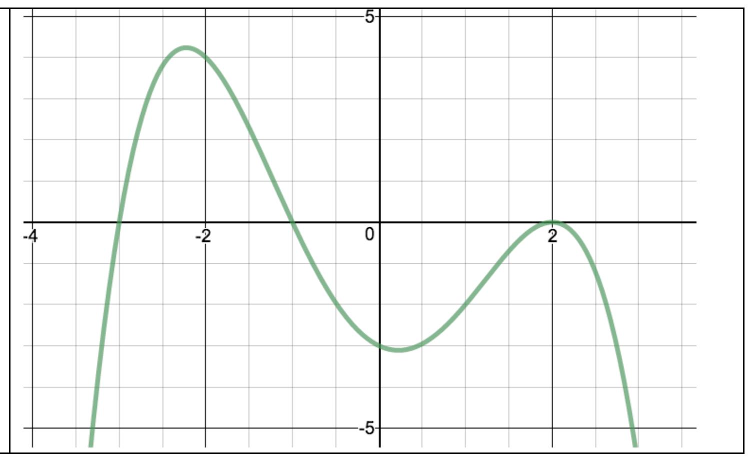
   c. Evaluate (i) $p(2)$ (ii) $p(4)$ (i) $p(-5)$

   d. Sketch the polynomial.

   e. Solve the inequality $p(x) < 0$, given your work in Question (d).

4. Now that you understand the connection between factors and roots as well as multiplicity of roots, write the equations of the following graphs of the polynomials both in factored and in standard form. $\{5\}$





5. Given these rational functions, use DESMOS & TI-84 to determine the: $\{1,4,7\}$

   a. equation of the vertical asymptote

   b. equation of the horizontal asymptote

   i. $f(x) = \dfrac{6x-7}{3x+4}$ ii. $f(x) = \dfrac{2x+7}{4x-8}$ ii. $f(x) = \dfrac{4x+5}{2x-3}$

   c. Finally, if the equation of a rational function was $f(x) = \dfrac{Ax+B}{Cx+D}$, state the equations of both the vertical and the horizontal asymptotes

6. The following exercise is based on the EXTREMA and ZEROS of a polynomial function. From the description of each polynomial function, sketch a possible graph of each function clearly showing the correct number of extrema or zeros. DO NOT repeat any graphs! If it is not possible to sketch a function fitting the criteria, EXPLAIN why. {1}
   a. 6^{th} degree polynomial with 4 x-intercepts
   b. 5^{th} degree polynomial with 1 extrema
   c. 4^{th} degree polynomial with 2 extrema
   d. 6^{th} degree polynomial with 2 x-intercepts.

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

1. Go online and research the "Factor Theorem". Then, practice factoring the following polynomials

(a) $f(x) = x^4 - 8x^3 + 24x^2 - 32x + 16$

(b) $f(x) = x^4 - 2x^3 - 3x^2 + 4x + 4$

(c) $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$

(d) $f(x) = x^4 - 8x^3 + 15x^2 + 4x - 20$

(e) $f(x) = x^4 - x^3 - 18x^2 + 16x + 32$

(f) $f(x) = x^4 + 14x^2 + 40$