BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. So, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook.

1. The table included shows the population of Thunder Bay from 1966 to 1998. Using x as the number of years since 1966, determine a quartic model for this data set. (17}

Year	1966	1976	1986	1996	1998
Population	143,673	119,253	122,217	125,562	128,607

a. Write the equation as $y = P(x)$ where each coefficient will be rounded to two decimal places.

b. Explain why Mr. S has decide to restrict the domain for this model to $\{x \in R \mid 0 \leq x \leq 32\}$.

c. Determine when the population was 122,775.

d. When was the population of Thunder Bay at a minimum? What was this minimum population.

2. To introduce one major algebraic tool for working with factorizing polynomials, please watch the following video dealing with Synthetic Division (https://youtu.be/5dBAdzl2Mns). Then, with the following questions, use synthetic division to simplify and state any remainder as a fraction:

a. $(2x^3 - 7x^2 - 7x + 19) \div (x - 1)$

b. $(6x^4 + 13x^3 - 34x^2 - 47x + 28) \div (x + 3)$

c. $(x^3 - 7x - 6) \div (x - 3)$

d. $(2x^3 + x^2 - 22x + 20) \div (x + 2)$

e. $(6x^3 - 2x - 15x^2 + 5) \div (x - 1)$

f. $(12x^4 - 56x^3 + 59x^2 + 9x - 18) \div (2x + 1)$

3. With this question, you will (hopefully) see the connection between (i) evaluating a polynomial and (ii) the remainder that results from a division. {11}

   a. Example #1
      i. Evaluate P(2) if $P(x) = x^2 - 6x - 7$
      ii. Divide $P(x) = x^2 - 6x - 7$ by $x - 2$ using synthetic division.
      iii. What is the remainder after the division?

   b. Example #2
      i. Evaluate P(2) if $P(x) = 4x^3 - 2x^2 - 6x - 1$
      ii. Divide $P(x) = 4x^3 - 2x^2 - 6x - 1$ by $x - 2$ using synthetic division.
      iii. What is the remainder after the division?

   c. Example #3
      i. Evaluate P(-3) if $P(x) = 2x^3 - x^2 - 7x + 6$
      ii. Divide $P(x) = 2x^3 - x^2 - 7x + 6$ by $x + 3$ using synthetic division.
      iii. What is the remainder after the division?

4. The following exercise is based on the EXTREMA and ZEROS of a polynomial function. From the description of each polynomial function, sketch a possible graph of each function clearly showing the correct number of extrema or zeros. DO NOT repeat any graphs! If it is not possible to sketch a function fitting the criteria, EXPLAIN why. {1}

   a. 5^{th} degree polynomial with 4 x-intercepts
   b. 5^{th} degree polynomial with 2 extrema
   c. 4^{th} degree polynomial with 2 x-intercepts
   d. 6^{th} degree polynomial with 3 extrema

5. Use DESMOS and your TI-84 to graph the parent function $f(x) = \frac{1}{x}$ and sketch it in your notes. Then, as a second function, graph the rational function $g(x) = \frac{1}{x+3} + 1$ and sketch it in your notes. Explain the form of the second equation can be called "transformational form." {4,7}

6. Convert the following rational function equations from transformation form to linear/linear form. In each case, list the transformations that were applied to the parent function of $y(x) = \frac{1}{x}$. (i.e. Write the following as ONE fraction (think adding fractions)): {4,7}

a. $y(x) = \frac{2}{x} + 3$ b. $y(x) = 4 - \frac{3}{x}$ c. $y(x) = \frac{1}{x-2} - 4$

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

1. Go online and research the "Remainder Theorem". Then, practice finding the remainder in the following divisions.

a. Find the remainder when $x^2 + 6x - 17$ is divided by $x - 1$.
b. Find the remainder when $4x^2 - x + 3$ is divided by $x + 2$.
c. Find the remainder when $2x^3 - x^2 - x - 1$ is divided by $x - 1$.
d. Find the remainder when $x^4 - x^3 - x^2 - x - 1$ is divided by $x - 3$.
e. Find the remainder when $x^4 - 5x^3 + x^2 - 10x - 5$ is divided by $2x + 3$.
f. Solve for k: When $x^3 + kx + 1$ is divided by $x - 2$, the remainder is -3.
g. Solve for k: When $2x^4 + kx^2 - 3x + 5$ is divided by $x - 2$, the remainder is 3.
h. When $x^3 + kx^2 - 2x - 7$ is divided by $x + 1$, the remainder is 5. What is the remainder when it is divided by $x - 1$?
i. When the polynomial $x^n + x - 8$ is divided by $x - 2$ the remainder is 10. What is the value of n ?

2. General Conclusions – Parent Functions and End Behaviour and Multiplicity and Zeroes and Extrema:

a. What is the general appearance of an even degree polynomial function? An odd degree polynomial function?
b. What are the generalized end behaviours of even degree polynomials?
c. What are the generalized end behaviours of odd degree polynomials?
d. How can you predict the maximum number of zeroes of a polynomial?
e. How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is even?
f. How do you predict the appearance of a function near the x-axis if the multiplicity of its zeroes is odd?
g. How can you predict the maximum number of extrema in a polynomial?