BIG PICTURE of this Unit

- How can we extend our algebra skills to interchange between standard and factored form of polynomial equations? (i.e. synthetic division, factoring)
- Can we use our new polynomial algebra skills in order to find a method for solving EVERY polynomial equation (especially those that don't factor?)
- How can use the equation of a polynomial to analyze for key features of a graph of a polynomial (i.e. end behavior, multiplicity of roots, optimal points, intervals of increase/decrease).
- When and how can polynomial functions be used to model real world scenarios?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. So, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS!!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. Use DESMOS as well as your TI-84 to graph $y = \frac{1}{x}$. Use your graph to answer the following questions: {1,4}

   a. Make a sketch of this function in your notes.
   b. Label the TWO key points (which are (1,1) and (-1,-1)) on this function and write them down.
   c. Write down the equations of the asymptote(s).
   d. State the domain and range.
   e. State the zero(s) (if they exist).
   f. State the extrema (if they exist).
   g. State the intervals of increase and decrease.
   h. Is the function continuous or discontinuous. Explain your anwer.

2. Use DESMOS as well as your TI-84 to graph and then compare (what do all the graphs and equations share in common?) What do you notice about the way an equation is written and the way a graph looks? Include labeled sketches in your notes. {16}

   a. $y = 2(x-1)$
   b. $y = 2(x-1)(x+1)$
   c. $y = 2(x-1)(x+1)(x+2)$
   d. $y = 2(x-1)(x+1)(x+2)(x-3)$

3. Evaluate the function $g(x) = \dfrac{1}{x+2}$ for the following values of x:

   (i) $x = 3$ (ii) $x = 8$ (iii) $x = 28$ (iv) $x = 58$ (v) $x = 98$

   a. What are you noticing happening with both the *x*-values and the *y*-values?

   (i) $x = -1$ (ii) $x = -1.5$ (iii) $x = -1.9$ (iv) $x = -1.99$ (v) $x = -1.999$

   b. What are you noticing happening with both the x-values and the y-values?

   c. Graph the function on your TI-84. Explain how the picture of the graph now helps you make sense of you answers to (a) and (c).

4. Given the polynomial function, $p(x) = 2x^3 - 3x^2 - 14x + 15$, answer the following questions: {1,4}

   a. Graph the function on DESMOS as well as your TI-84 calculator.
   b. Find the zeroes.
   c. Given these zeroes, write the equation in factored form.
   d. Go to wolframalpha and factor this equation. How does it compare to your factors from part c?
   e. Go on line to find out what a "local maximum" is. In your notebooks, record your understanding of this term. Find the local maximum of this function.
   f. Go on line to find out what a "local minimum" is. In your notebooks, record your understanding of this term. Find the local minimum of this function.
   g. Go on line to find out what it means for a function to be "increasing". On what domain interval are the function values increasing?
   h. Go on line to find out what it means for a function to be "decreasing". On what domain interval are the function values decreasing?
   i. Go on line to find out what "end behaviour" means. State the end behaviours of this function.

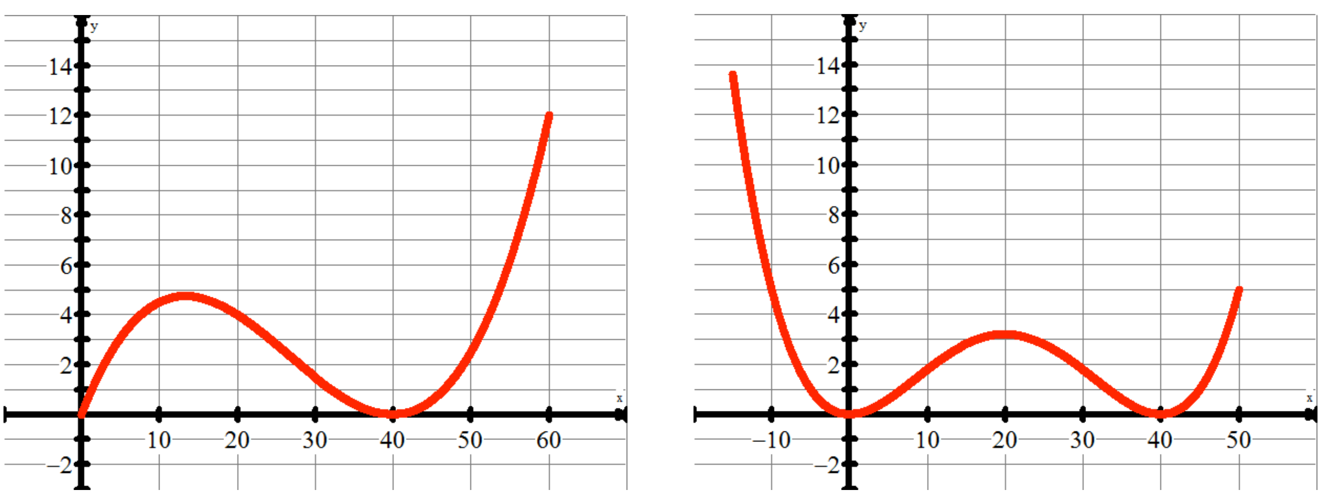
5. Use DESMOS as well as your TI-84 to graph these following "parent" functions. Include a labeled sketch in your notes: (i) $f(x) = x^3$ (ii) $g(x) = x^4$ (iii) $y = \dfrac{1}{x}$. Label the "important" features/parts of each function. {4}

6. SureGrip Athletic shoes tracks the relationship between total sales of shoes and the advertising expenses. The function used to model the relationship is $S(d) = -\dfrac{1}{4{,}000}d^3 + \dfrac{3}{20}d^2$, where S is shoes sales in millions of dollars and d is the expenses, measured in tens of thousands of dollars. {4,8,17}

   a. Explain what the ordered pair $S(200) = 4000$ means in the context of this model.
   b. Graph the function, given that the domain for this problem is restricted to $\{d \in R \mid 50 \le d \le 500\}$.
   c. What advertising expense (cost) would optimize the sale of shoes? What would be the optimal sales of shoes (revenues)?
   d. If the company wants to have sales of 6.5 billion dollars (S = 6500), how much should they spend on advertising expenses?
   e. In which domain interval are the sales decreasing? What might this mean for the company?

7. Roller Coaster Design {1,4,17}

   Your design team has the following "profile" of part of a roller coaster (the x-axis represents horizontal distance and the y-axis represents vertical distance). Your initial task is to find a polynomial model for the profile of the roller coaster.

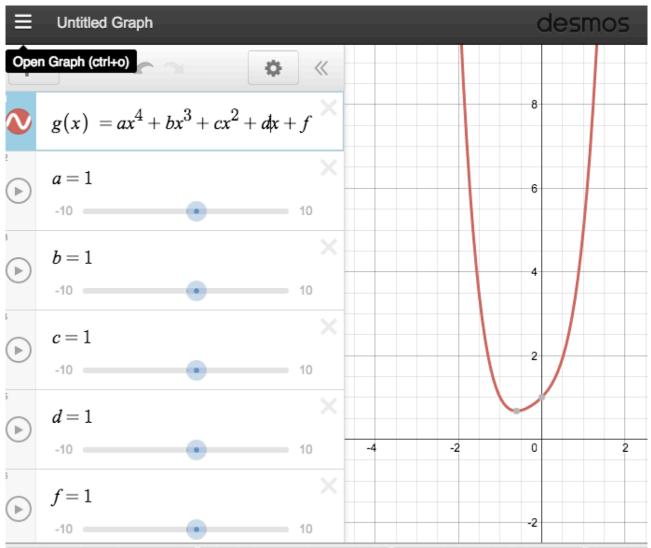


You will carry out this modeling in three ways:

(a) Using DESMOS, program in the standard form of a cubic equation ($y = ax^3 + bx^2 + cx + d$) or quartic ($y = ax^4 + bx^3 + cx^2 + dx + g$) and add sliders for values of a,b,c,d and g. Then adjust the sliders to get an equation that matches this "profile" pictured here. (see picture included for setting up sliders)

(b) Use the graph to read data points from the graph, then your TI-84 to determine the equation (cubicreg/quartreg)

(c) (NO CALCULATORS – ALL students should learn how to do this) Use algebra & skills you've learned from your Quadratics Unit to determine an equation. (HINT: factors and x-intercepts)



Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Polynomial and Rational Functions.

Use DESMOS to conduct the following investigation:

1. Graph the following functions: (i) $f(x) = x$ (ii) $f(x) = x^3$ (iii) $f(x) = x^5$ (iv) $f(x) = x^7$.
2. Explain what the function "looks like" when the exponent is odd (i.e. x^{2n+1}, where $n \geq 0$)
3. Graph the following functions: (i) $f(x) = x^2$ (ii) $f(x) = x^4$ (iii) $f(x) = x^6$ (iv) $f(x) = x^8$.
4. Explain what the function "looks like" when the exponent is even (i.e. x^{2n}, where $n \geq 1$)