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|---|---|
| BIG PICTURE of this UNIT: | • How do we WORK WITH & EXTEND the concept of "functions"<br>• Why are quadratic equations written in different forms?<br>• How do we EXTEND and APPLY our knowledge of quadratic functions, beyond the basics of IM2? |

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. So, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS!!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. (CI) To analyze the parabola defined by $f(x) = -\frac{1}{2}(x+4)^2 + 12$: {3,5,6}
   a. State the equation of the axis of symmetry
   b. State the location of the vertex
   c. State the domain interval in which the function values are **increasing**.
   d. Evaluate f(0)
   e. Evaluate f(-7)
   f. State the location of a point "symmetrically opposite" to the y-intercept
   g. Solve f(x) = -60
   h. Solve for x if f(x) = 0. Round your answer(s) to two decimal places
   i. Write the equation of the INVERSE function of f(x) and write it in $f^{-1}(x)$ notation

2. The length of a rectangle is 7 units more than its width. If the width is doubled and the length is increased by 2, the area is increased by 42 square units. Find the dimensions of the original rectangle. {2,9,17}

3. Quadratic – Linear Systems: {7,9,12}
   a. Find the intersection point(s) of the parabola $y = x^2 + 4x - 7$ and the line $y = 3x - 1$.
   b. Find the intersection point(s) of the parabola $y = 2x^2 + 4x + 4$ and the line $y = -8x - 14$.
   c. Find the intersection point(s) of the parabolas $y = (x - 2)(x + 3)$ and $y = -(x + 1)^2+15$

4. (CI) A company's profit, $P(c)$, in thousands of dollars, on sales of computers is modelled by the function $P(c) = -2(c-3)^2 + 50$, where $c$ is in thousands of computers sold. {3,15,16}

   a. Explain what $P(1) = 42$ means in the context of this problem.

   b. What is the maximum profit that the company can reach when they sell computers.

To increase their profitability, the company sells anti-virus software. Their profit, in thousands of dollars, on sales of antivirus software is modelled by the function $P(v) = -2(v-2)(v-8)$, where $v$ is in thousands of software packages sold.

   c. Explain what $P(6) = 16$ means in the context of this problem.

   d. What is the maximum profit that the company can reach when they sell software packages.

   e. HL EXTENSION: As a promotional offer, the company now puts the computers and anti-virus software together as a "special package" and reduces the price of the total package by $75. Assuming that the profit models remain the same (i.e. $P(c)$ and $P(v)$ ), determine the maximum profit that the company can earn.

5. (CI) Using the equation $f(x) = 2(x+4)^2 - 8$, {5,9}

   a. determine the x-intercepts by using "inverse operations" to solve $f(x) = 0$.

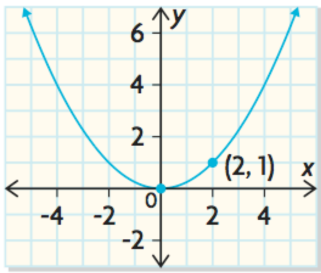
   b. determine the equation of the inverse of $f(x) = 2(x+4)^2 - 8$, again using "inverse operations."

   c. Verify your inverse equation using DESMOS. Is the inverse of this quadratic function a function? Explain.

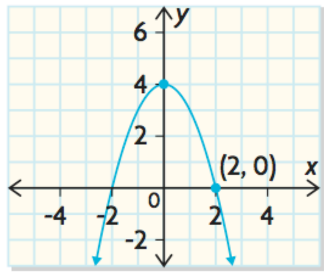
6. Write the equation of each parabola graphed below. Show the key steps of your solution. {3,6}

Write the equation of each parabola in vertex form.

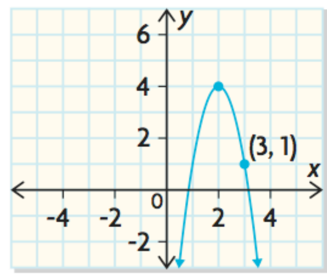
**a)**




**c)**




**e)**

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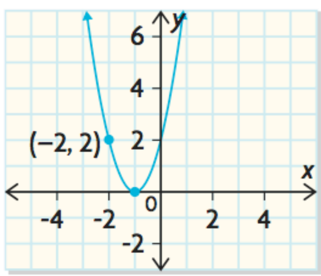


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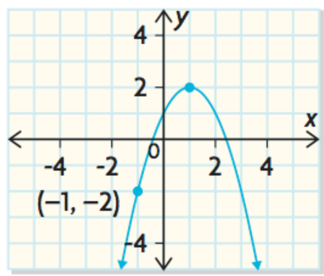
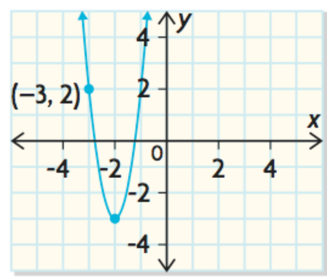



**f)**




7. Given the following equations: {5,8}

   a. TASK A: Convert between forms of equations:

   b. TASK B: From each parabola in the left column, determine HOW it was transformed from $y = x^2$.

   c. TASK C: Write the equation of the INVERSE RELATION of each parabola in the left column. Graph it on DESMOS

| Vertex to Standard | Standard to Vertex (HINT: x = -b/2a) |
|---|---|
| $y = (x - 5)^2 + 2$ | $y = x^2 + 4x - 12$ |
| $y = -3(x + 4)^2 + 24$ | $y = 2x^2 - 20x - 80$ |
| $y = -\frac{1}{2}(x + 10)^2 - 22$ | $y = 4x^2 + 20x + 25$ |

**Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Quadratic Functions.**

1. Solving Quadratic Inequalities: To solve $(x+2)(x-1) > 0$ we need to set up two conditions: {26}

   Condition #1 is that BOTH (x + 2) and (x – 1) are positive numbers, so BOTH (x + 2) > 0 AND (x – 1) > 0 ➔ so for what values of x is this true? ➔ ONLY if x > 1 will BOTH expressions be positive

   Condition #2 is that BOTH (x + 2) and (x – 1) are negative numbers, so BOTH (x + 2) < 0 AND (x – 1) < 0 ➔ so for what values of x is this true? ➔ ONLY if x < -2 will BOTH expressions be negative

   a. EXPLAIN why conditions #1 and #2 need to be true.

   b. Now, solve the following quadratic inequalities by factoring:

      (i) $(x + 3)(2x - 1) \geq 0$ (ii) $(2x - 3)(x + 6) \leq 0$

      (iii) $2x^2 - 14x > -20$ (iv) $3x^2 - 4 \leq x$

2. Solving Quadratic Inequalities: To solve $x^2 + x - 2 > 0$ without factoring, we can use the method of completing the square. {26}

   a. Show that $x^2 + x - 2 = \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$.

   To solve $\left(x + \frac{1}{2}\right)^2 - \frac{9}{4} > 0$ or $\left(x + \frac{1}{2}\right)^2 > \frac{9}{4}$, we use square roots ➔ $\sqrt{\left(x + \frac{1}{2}\right)^2} > \sqrt{\frac{9}{4}}$ and then absolute value, so we then get $\left|x + \frac{1}{2}\right| > \frac{3}{2}$

   b. Therefore, now solve $\left|x + \frac{1}{2}\right| > \frac{3}{2}$

   c. Now, solve the following quadratic inequalities:

      (i) $(x + 4)^2 - 2 < 23$ (ii) $\frac{1}{2}(x+3)^2 - 9 \geq -1$ (iii) $\frac{1}{3}(x-1)^2 - 4 < 5$

   d. Using the method of completing the square to solve the following quadratic inequalities:

      (i) $x^2 + 2x - 5 > 0$ (ii) $x^2 - 3x - 8 > 1$ (iii) $3x^2 - 4x + 5 < 0$