BIG PICTURE of this UNIT:	• How can we visualize events and outcomes when considering probability events? • How can we count outcomes in probability events? • How can we calculate probabilities, given different types of events • Can we predict how likely it is that an event occurs? How can we use that knowledge?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. The questions will involve basic ideas including (i) visualizing the outcomes of probability events/experiments, (ii) determining probabilities of single and compound probability events, (iii) counting outcomes, and (iv). EVERY PROBLEM SET will involve spiralling through these major concepts as you will be given the opportunity to deepen and extend your conceptual knowledge & skill set on these major themes as you see them multiple times in our lessons.

So, in your group, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. Go on line and find out the difference between compound events that are INDEPENDENT and DEPENDENT. Then decide whether the following events are independent or dependent?

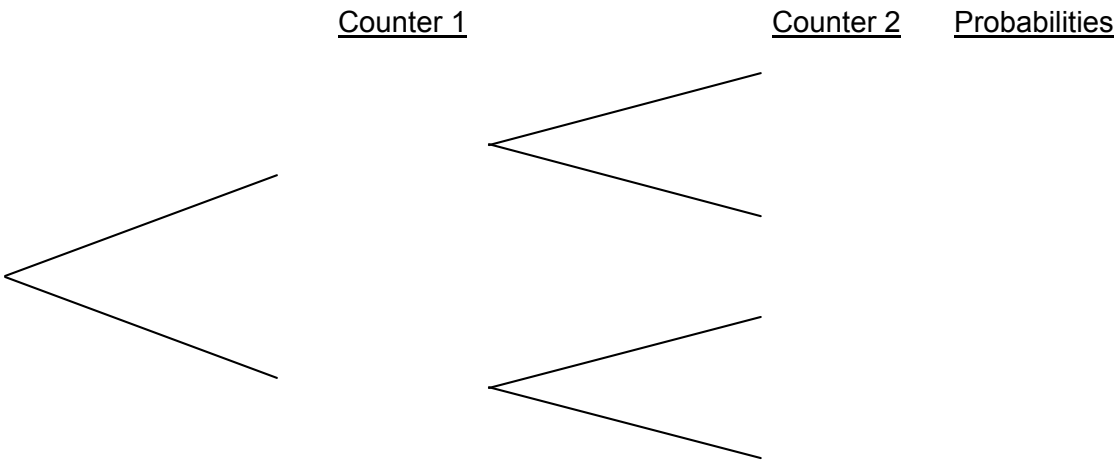
   a. Roll a die; toss a coin
   b. Take a marble out of a bag; take a second marble out of a bag.
   c. Choose a person from a group of 50 persons. Choose another person from the same group.
   d. Draw a card from the deck and put it back. Draw a card again from the same deck.
   e. You have a jar with 24 pieces of chocolate candy and 14 pieces of orange candy. We take one piece of candy at random from the jar, put it back, and then take a second piece of candy at random from the jar.
   f. Reem has a blue, red, and green tie. She also has a blue and green shirt. Reem chooses a random tie and shirt for work today.
   g. Yeong Kwon plays card games. He picks a card at random. Then without putting the first card back, he picks a second card at random.
   h. Kyumin has 14 coins. He takes 3 of them at random, then he puts these back, and then pick 2 more coins at random.
   i. Mazzin has $4,700 in his bank account. He withdraws $1,200 from his bank account to pay for rent. Brett books a vacation 3-days later that costs $4,000. He withdraws $3,500 from his account and goes on a payment pay for the remainder.
   j. Hoda has 10 handmade sheets. She takes 6 sheets at random. Then without putting these sheets back, she picks 2 sheets at random.
   k. Jeff has 3 children. His first 2 children are boys. His last child is a girl.

2. License plates have 3 letters followed by 4 numbers.

   a. If the same letter or number can be repeated, how many can be made?

   b. If the same letter CANNOT be repeated, how many can be made?

3. A bag contains 7 red counters and 4 blue counters. A disc is taken at random from the bag and not put back in. A second disc is then removed from the bag. Complete the tree diagram below:



Find the probability that:

a. Both discs are red b. Both discs are blue c. Both discs are different

4. Take a card from a normal deck, without looking at it.

   a. What is the probability you drew a spade?_____

   b. Now assume that you peeked just a little and know that the card is black. What is the probability you drew a spade?_____

   c. Assume you didn't peek but your friend did and tells you that the card is not a heart. What is the probability you drew a spade?_____

   d. The proper notation for the last question is P(spade | not a heart). This means "the probability of a spade given that the card is not a heart." The symbol for a known or given condition is a _____ and the given fact determines the _____of the probability.

5. A Class of 40 students completed a survey on what pets they like. The choices were: Cats, Dogs, and Birds. Everyone liked at least one pet.

10 students liked Cats and Birds but not dogs

2 students liked Dogs and Birds but not Cats

10 students liked Cats only

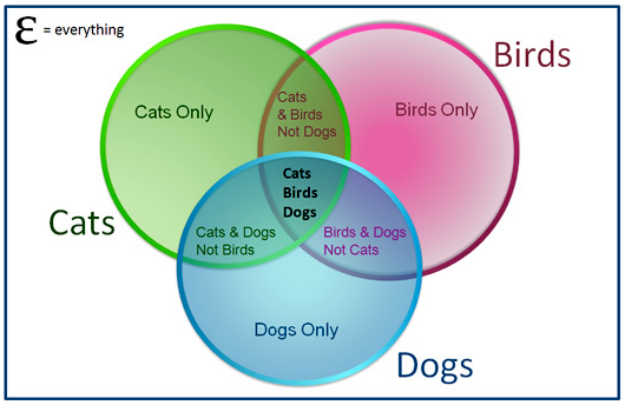
1 student liked Birds only

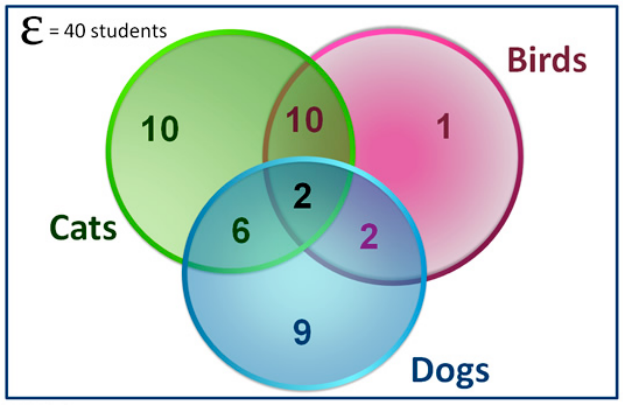
6 students liked Cats and Dogs but not birds

2 students liked all three pets

9 students liked Dogs only

Represent these results using a three circle Venn Diagram.





How probable is it that a randomly selected student:

a. has a bird;

b. has a bird only;

c. has a bird or a dog

d. has a bird and a dog

e. has a bird given that they have a dog

f. has a dog given that they have a bird

g. has a dog, but not a cat

h. has either a dog or a cat, but not a bird

i. has neither a dog nor a cat

6. One marble is randomly drawn and then replaced from a jar containing two white marbles and one black marble. A second marble is drawn. What is the probability of drawing a white and then a black?

7. One bag contains 2 green marbles and 4 white marbles, and a second bag contains 3 green marbles and 1 white marble. If Trent randomly draws one marble from each bag, what is the probability that they are both green?

8. On a certain day the chance of rain is 80% in San Francisco and 30% in Sydney. Assume that the chance of rain in the two cities is independent. What is the probability that it will NOT rain in either city? (It will not rain in BOTH cities.)

9. A club has 25 members, 20 boys and 5 girls. Two members are selected at random to serve as Executive Officers (i.e president and vice president.) What is the probability that both executive officers will be girls.

Higher Level Questions for More Complex Concepts in Probability. Determine the probability of the event described in each exercise. Unless stated otherwise, assume all items of chance (dice, coins, cards, spinners, etc.) are fair.

1. Here are some problems dealing with a concept called "Expected Value"

EXAMPLE 5 **Expected Value as Average Payoff**

A game is played using one die. If the die is rolled and shows 1, 2, or 3, the player wins nothing. If the die shows 4 or 5, the player wins $3. If the die shows 6, the player wins $9. If there is a charge of $1 to play the game, what is the game's expected value? Describe what this means in practical terms.

EXAMPLE 6 **Expected Value and Roulette**

One way to bet in roulette is to place $1 on a single number. If the ball lands on that number, you are awarded $35 and get to keep the $1 that you paid to play the game. If the ball lands on any one of the other 37 slots, you are awarded nothing and the $1 that you bet is collected. Find the expected value for playing roulette if you bet $1 on number 20. Describe what this means.

1.2 Some Exercises for You

Determine the expected value for the games.

1. Charge \$1 to play. Roll one die, with payouts as follows:

Roll	Payout
6	\$ 2
5	\$ 2
4	\$ 1
3	\$ 0
2	\$ 0
1	\$ 1.50

2. Charge: \$1 to toss 3 coins. Toss the coins. If you get all heads or all tails, you receive \$5. If not, you get nothing.
3. Charge: \$1. Roll 2 dice. If you roll 2 odd numbers, like a 3 and a 5, you get \$2. If you roll 2 even numbers, like 4 and 6, you get \$2. Otherwise, you get nothing.
4. Charge: \$5. Draw twice from a bag that has one \$10 and 4 \$1 bills. You get to keep the bills.