

BIG PICTURE of this UNIT:

- How do we WORK WITH & EXTEND the concept of “functions”
- Why are exponential equations written in different forms?
- How do we EXTEND our knowledge of exponential functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. The questions will involve basic ideas from IM2 including (i) exponential functions, (ii) functions in general, (iii) linear functions, and (iv) number patterns. EVERY LESSON this semester will involve spiralling through these 4 major concepts as you will be given the opportunity to deepen and extend your conceptual knowledge & skill set on these 4 major themes as you see them multiple times in our lessons.

So, in your group, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. Four months ago after it stopped advertising, a manufacturing company noticed that its sales per unit “y” had dropped each month according to the function $y = 100,000e^{-0.05x}$ where “x” is the number of months after the company stopped advertising. {4,11,20}
 - a. Find the projected drop in sales per unit six months after the company stops advertising.
 - b. How many months until sales per unit had dropped \$50,000?
2. Solve the following logarithmic equations: {6,7}
 - a. SL level (<http://www.mathworksheets4kids.com/logarithms/solve-level2-easy1.pdf>)
 - b. HL Level (<http://www.mathworksheets4kids.com/logarithms/evaluating-expressions-level2-hard1.pdf>)
3. At any time $t \geq 0$, in days, the number of bacteria present, y, is given by $y(t) = Ce^{kt}$ where k is a constant. The initial population is 1000 and the population triples during the first 5 days. {4,11,20}
 - a. What does “C” represent? What is its value?
 - b. By what factor will the population have increased in the first 10 days?
 - c. At what time, t in days, will the population have increased by a factor of 6?

4. (CI) Evaluate the following: {6,7}

$$\begin{array}{cccccc} \log_4 64 = & \log_2 32 = & \log_{\frac{1}{5}} 25 = & \log_{12} 144 = & \log_4 2 = & \log_{\frac{2}{3}} \left(\frac{4}{9} \right) = \\ \log_{125} 5 = & \log_9 3 = & \log_8 2 = & \log_2 \frac{1}{16} = & \log_{243} 27 = & \log_8 4 = \\ \log_2 \frac{1}{8} = & \log_9 \frac{1}{81} = & \log_3 \frac{1}{27} = & \log_{\frac{3}{5}} \left(\frac{25}{9} \right) = & \log_{27} \frac{1}{3} = & \log_{128} \frac{1}{2} = \end{array}$$

5. During a certain epidemic, the number of people that are infected at any time increases at a rate proportion to the number of people that are infected at that time. If 1000 people are infected when the epidemic is first discovered and 1200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered? {4,11,20}

6. The following data represents the price and quantity supplied in 1997 for IBM personal computers. {4,11,20}

| | | | | | | | |
|---------------------|------|------|------|------|------|------|------|
| Price (\$/Computer) | 2300 | 2000 | 1700 | 1500 | 1300 | 1200 | 1000 |
| Quantity Supplied | 180 | 173 | 160 | 150 | 137 | 130 | 113 |

- Use your calculator draw the scatter plot. Use price as the independent variable. Label your axes.
- Use your calculator to fit a logarithmic model to this data.

7. Let $P(t)$ represent the number of wolves in a population at time t years, when $t \geq 0$. The population of wolves is given by $P(t) = 800 - P_0 e^{-kt}$. {4,11,20}

- If $P(0) = 500$, find $P(t)$ in terms of t and k .
- If $P(2) = 700$, find k .
- As time increases without bound, what happens to the population of wolves? Support your answer with a graph and a written explanation.



Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Exponential and Logarithmic Functions.

1. Investigate the behaviour of the functions $g_1(x) = xe^x$, $g_2(x) = x^2e^x$, and $g_3(x) = x^3e^x$ as $x \rightarrow \infty$ and as $x \rightarrow -\infty$ and find any horizontal asymptotes. Generalize to functions in the form of $g_n(x) = x^n e^x$, where n is any positive integer.
2. Investigate the behaviour of the functions $g_1(x) = x/e^x$, $g_2(x) = x^2/e^x$, and $g_3(x) = x^3/e^x$ as $x \rightarrow \infty$ and as $x \rightarrow -\infty$ and find any horizontal asymptotes. Generalize to functions in the form of $g_n(x) = x^n/e^x$, where n is any positive integer.
3. It is common practice in many applications of mathematics to approximate non-polynomial functions with appropriately selected polynomials. For example, the polynomials in this problem are called Taylor Polynomials are being used to approximate the exponential function $f(x) = e^x$. To illustrate this approximation graphically, use DESMOS as well as your TI-84 to draw the exponential function, $f(x) = e^x$ as well as the polynomial in the viewing window $-4 < x < 4$ and $-5 < y < 50$

$$P_1(x) = 1 + x + \frac{1}{2}x^2$$

$$P_2(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

$$P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$$

$$P_4(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5$$