BIG PICTURE of this UNIT:	• How do we WORK WITH & EXTEND the concept of "functions" • Why are exponential equations written in different forms? • How do we EXTEND our knowledge of exponential functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. The questions will involve basic ideas from IM2 including (i) exponential functions, (ii) functions in general, (iii) linear functions, and (iv) number patterns. EVERY LESSON this semester will involve spiralling through these 4 major concepts as you will be given the opportunity to deepen and extend your conceptual knowledge & skill set on these 4 major themes as you see them multiple times in our lessons.

So, in your group, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. (CI) Evaluate the following expressions: {1,3,6,7}

(a) $\log_3(81)$ (b) $\log_6\left(\frac{1}{216}\right)$ (c) $16^{\frac{3}{2}}+16^{-0.5}+64^{\frac{1}{2}}-27^{\frac{2}{3}}$

(d) $81^{\frac{1}{2}}+\sqrt[3]{8}-32^{\frac{4}{5}}+16^{\frac{3}{4}}$ (e) $\log_{81}(9)$ (f) $\log_3(\sqrt[3]{9})$

(g) $\log_{\frac{1}{2}}(16)$ (h) $\sqrt[3]{-27}+\left(\sqrt[3]{-216}\right)^5+\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$ (i) $\log_{25}\left(\frac{1}{125}\right)$

2. A hard-boiled egg has a temperature of 98 degrees Celsius. If it is put into a sink that maintains a temperature of 18 degrees Celsius, its temperature x minutes later is given by the formula $T(x)=18+80e^{-0.29x}$. Hilda needs her egg to be exactly 30 degrees Celsius for decorating. How long should she leave it in the sink? {4,11,20}

3. Given the functions of $f(x) = -\frac{1}{2}(x-4)$ and $g(x) = 9\left(\frac{1}{3}\right)^{x+2}$, determine: {9,10,13,14}

   a. Equations for $(i)\ f \circ g(x) \qquad (ii)\ g \circ f(x)$

   b. The domain and range for the functions defined by $(i)\ f \circ g(x) \qquad (ii)\ g \circ f(x)$

   c. The intercepts for the functions defined by $(i)\ f \circ g(x) \qquad (ii)\ g \circ f(x)$

   d. Equations for the inverses of $f^{-1}(x)$ and $g^{-1}(x)$.

   e. The solution to the inequality $f^{-1}(x) > g(x)$.

4. (CI) Given the function $f(x) = 8 - 2^{x+4}$; {8,9,13,16,17}

   a. Determine the domain, range, asymptote(s) and intercept(s) of $f(x)$. Sketch and label key points.

   b. The function $f(x) = 8 - 2^{x+4}$ represents a transformation of the "parent function" $y = 2^x$. Describe which transformations must be applied to $f(x) = 8 - 2^{x+4}$ to get back to $y = 2^x$.

   c. Determine the equation of the inverse of $f^{-1}(x)$. Sketch and label key points.

   d. (CA) Use your TI-84 to graph $y = |f(x)|$ (that is $y = \left|8 - 2^{x+4}\right|$. Sketch it in your notes. Explain WHY the graph now appears as it does.

   e. (HL) Determine the equation of $f^{-1} \circ f(x)$ and $f \circ f^{-1}(x)$. Show the key steps of your work. Expain the significance of the result.

5. Questions dealing with half-life can use a "special equation/formula". Go on line and find this formula. Use this formula to answer the following questions: {11,20}

   a. The half life of caffeine in a child's system when a child eat or drinks something with caffeine is 1.5 hours. How much caffeine would remain in a child's body if the child ate a chocolate bar with 30 mg of caffeine 8 hours before?

   b. The half-life of Carbon-14 is about 5370 years. What percentage of the original carbon-14 would you expect to find in a sample after 2500 years?

   c. Now rework these questions, wherein we now set up the scenario of continuous changes, hence you must use the special base of e. So, write new equations using base e to rework your solutions.

6. There has been an accident at the local nuclear plant and a new radioactive material (Santogen) has been spilled. This radioactive material begins to decay exponentially. Assume that this decay is continuous. There were 1820 Bq (becquerels) of Santogen initially. Eight hours later there were 576 Bq. {11,20}

   a. What is the decay rate of Santogen?
   b. This material becomes non-leathal when there is a max of 20 Bq. When will it be safe for workers to enter the space and clear it out?
   c. Will there ever be 0 Bqs left of the Santogen material?
   d. What is the half life of Santogen?

7. Use your TI-84 to determine the value of the following logarithms: {6,7}

$\log_2 0$	$\log_2 1$	$\log_2 2$	$\log_2 3$	$\log_2 4$	$\log_2 5$	$\log_2 6$
$\log_2 7$	$\log_2 8$	$\log_2 9$	$\log_2 10$	$\log_2 11$	$\log_2 12$	$\log_2 13$
$\log_2 14$	$\log_2 15$	$\log_2 16$	$\log_2 17$	$\log_2 18$	$\log_2 19$	$\log_2 20$

Look for patterns amongst the numbers & outputs:

   a. Compare $\log_2 2$ and $\log_2 5$ and $\log_2 10$
   b. Compare $\log_2 3$ and $\log_2 4$ and $\log_2 12$
   c. Compare $\log_2 3$ and $\log_2 6$ and $\log_2 18$

   d. Can you see some patterns that will lead to some GENERALIZATIONS that would then in turn allow us to make PREDICTIONS?

      i. So, predict the value of (i) $\log_2 48$, (ii) $\log_2 36$, (iii) $\log_2 75$
      ii. So, predict the value of (i) $\log_2\left(\frac{1}{3}\right)$, (ii) $\log_2 7.5$, (iii) $\log_2 \sqrt[3]{12}$

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Exponential and Logarithmic Functions.

1. Working with the population function $P(t) = \dfrac{A}{1 + Be^{-0.2t}}$, it is known that $P(0) = 4$ and $P(10) = 23.54$. Determine the values of *A* and *B*.

2. Solve the following equations algebraically: (i) $\dfrac{e^{x} + e^{-x}}{e^{x} - e^{-x}} = 3$ and then (ii) $e^{2x} - 5e^{x} + 6 = 0$.