BIG PICTURE of this UNIT:	• How do we WORK WITH & EXTEND the concept of "functions" • Why are exponential equations written in different forms? • How do we EXTEND our knowledge of exponential functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. The questions will involve basic ideas from IM2 including (i) exponential functions, (ii) functions in general, (iii) linear functions, and (iv) number patterns. EVERY LESSON this semester will involve **spiralling through** these 4 major concepts as you will be given the opportunity to deepen and extend your conceptual knowledge & skill set on these 4 major themes as you see them multiple times in our lessons.

So, in your group, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. A formula that you can use for "continuous compounding" or "continuously changing" is $A(t) = A_0 e^{rt}$. Use this formula to answer these two questions about an investment I made wherein I invested \$10,000 in a fund yielding 12% p.a compounded continuously. {4,11,20}

    a. Find the value of the investment after 5 years.
    b. How long does it take for the investment to triple in value?

2. Determine the value after 2 years of a \$1000 investment invested at 10% pa under the following compounding conditions: {21}

    a. 10% pa compounded quarterly
    b. 10% pa compounded monthly
    c. 10% pa compounded **continuously**
    d. Explain why the three values you calculated are ***not the same***.

3. You invest 6,500 Euro in a bank that gives you a 5.5% annual interest rate, **compounded continuously**. {11,20}

    a. Create an equation to model the growth of your principle investment.
    b. What is your investment worth after 7 years?
    c. How long will it take to triple your investment?
    d. If you keep the same interest rate, and want to have 30,000 Euro after 20 years… how much should your Principle Investment be? Round to the nearest dollar.

4. Solve and provide approximate (**CA**) and exact solutions (**CI**) for the following equations. Solutions should be BOTH algebraic and graphic (to verify): {4,5,10}

(a) Solve $2^x = 7$ (b) Solve $4 + 2^x = 7$ (c) Solve $2^{x-2} = 7$

(d) Solve $3(2^x) = 7$ (e) Solve $2^{3x} = 7$ (f) Solve $1 - 2^{3(x-2)} = -7$

5. (**CI**) Evaluate & solve the following logarithmic expressions/equations. {6,7}

Evaluate the following logarithmic expressions	Solve the following logarithmic equations
$\log_5 125 =$	$\log_x 32 = 5$
$\log_2 \dfrac{1}{16} =$	$\log_3 x = 3$
$\log_2 \dfrac{1}{128} =$	$\log_3 81 = x$
$4\log_9 3 =$	$\log_5 x = -2$
$\log_4 256 =$	$\log_6 x = 2$

6. (CI – ideally) For the following functions, determine {8,9,13}

a. the equations of their inverses:
b. the domain and ranges of BOTH the function and its inverse
c. Include a sketch of the function and its inverse.

(i) $f(x) = 2^x$ (ii) $f(x) = \left(\dfrac{1}{2}\right)^x$

(iii) $f(x) = 3(2)^x$ (iv) $f(x) = 3 + 2^x$ (v) $f(x) = 2^{x+3}$

7. A sequence of numbers is simply a list of numbers to which there may or not be a given pattern. {22}

   a. Go online and explain what an arithmetic sequence is.
   b. Go online and explain what a geometric sequence is.
   c. Classify the following sequences as either arithmetic or geometric or neither
      i. 5, 15, 45, 135,….
      ii. 3, 9, 81, 6561,….
      iii. 15, 26, 37, 48, …..
      iv. 6000, 3000, 1500, 750, …..
   d. Determine the 10^{th} term in each sequence.

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Exponential and Logarithmic Functions.

1. (CA) Graph these functions: $(i)\ y = \dfrac{1}{1+e^{-x}}$ $\quad (ii)\ y = e^{-x^2}$ $\quad (iii)\ y = e^{x} + e^{-x}$ $\quad (iv)\ y = 1 - e^{-x}$

2. Given the function $g(x) = e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$,

   a. Determine the value of $f(0)$.
   b. Determine the "end behaviour" as $x \to +\infty$ (say, "evaluate" $f(1000)$).
   c. Determine the "end behaviour" as $x \to -\infty$ (say, "evaluate" $f(-1000)$).
   d. Is $g(x) = e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$ an even/odd/neither function? Explain how you know.
   e. Graph $g(x) = e^{-\frac{1}{2}x} + e^{\frac{1}{2}x}$

3. Given the function $g(x) = e^{-x^2}$,

   a. Determine the value of $f(0)$.
   b. Determine the "end behaviour" as $x \to +\infty$ (say, "evaluate" $f(1000)$).
   c. Determine the "end behaviour" as $x \to -\infty$ (say, "evaluate" $f(-1000)$).
   d. Is $g(x) = e^{-x^2}$ an even/odd/neither function? Explain how you know.
   e. Graph $g(x) = e^{-x^2}$