BIG PICTURE of this UNIT:	• How do we WORK WITH & EXTEND the concept of "functions" • Why are exponential equations written in different forms? • How do we EXTEND our knowledge of exponential functions, beyond the basics of IM2?

This lesson will be based upon a STUDENT DIRECTED DISCUSSION model ….. in your groups, you should be having DISCUSSIONS about how to think and work through and then present the solutions to the following questions. The questions will involve basic ideas from IM2 including (i) exponential functions, (ii) functions in general, (iii) linear functions, and (iv) number patterns. EVERY LESSON this semester will involve **spiralling through** these 4 major concepts as you will be given the opportunity to deepen and extend your conceptual knowledge & skill set on these 4 major themes as you see them multiple times in our lessons.

So, in your group, discuss & prepare solutions to the following questions. Record the key ideas of your discussions/solutions in your notebook. Then, once you have had your discussions, present your solutions on the board. Solutions do NOT necessarily NEED to be correct – they simply form the basis for DISCUSSIONS !!!! If your group has (i) multiple solutions that lead to the same answers OR (ii) same/different solutions that lead to different answers, present them ANYWAY!!

1. Do NOT use a calculator as you work these out {1,4,13}

Evaluate $4^{\frac{3}{2}}$ Evaluate $f^{-1}\left(\frac{1}{4}\right)$ if $f(x)=2^x$ Evaluate $f\left(\frac{3}{5}\right)$ if $f(x)=32^x$ Evaluate $f\left(-\frac{3}{2}\right)$ if $f(x)=9^x$

Evaluate $\left(\frac{16}{81}\right)^{\frac{3}{4}}$ Evaluate $g\left(\frac{3}{2}\right)=\left(\frac{1}{4}\right)^{-x}$ Evaluate $f^{-1}\left(\frac{1}{4}\right)$ if $f(x)=(-8)^x$ Evaluate $\left(\frac{-343}{1000}\right)^{\frac{2}{3}}$

2. Use GEOGEBRA to graph $f(x)=2^x$ and then perform the following transformations: {14,17,18,19}
   a. Reflect $f(x)$ across the y-axis and determine the new equation of this function.
   b. Reflect $f(x)$ across the x-axis and determine the new equation of this function.
   c. Reflect $f(x)$ across the line y = x and determine the new equation of this function.
   d. Translate $f(x)$ five units right and up 4 and determine the new equation of this function.
   e. If $g(x)=x-3$, PREDICT the appearance of the graphs of: $(i)\ f\circ g(x)$ $(ii)\ g\circ f(x)$.

3. (CI) Solve the following equations and verify using your TI-84: {4,10}

   (a) Solve $3(2^x)=24$ (b) Solve $3-(2^x)=-29$ (c) Solve $9^{-x-2}=\left(\frac{1}{27}\right)^{x+3}$

4. (**CA**) Working with a **new base**. The number of bacteria in a culture is given by the function $N(t) = 10e^{0.22t}$, where t is time in hours and $t > 0$. {11,20}

   a. What is the initial population of the culture (at $t = 0$)?
   b. Evaluate and interpret $N(15)$.
   c. Solve and interpret $500 = N(t)$.
   d. What is the doubling time for this bacterial population?
   e. What is the rate of growth of this bacterium population? Express your answer as a percentage.

5. (CA) Given the following sequence of numbers, identify the pattern present in the sequence and then use this pattern to predict the 10th term: {22}

   a. 5, 3.75, 2.8125, 2.109375, 1.58203125 , …….. (CHALLENGE: Find the SUM of the first ten terms, first 10,000 terms, the sum of an infinite number of terms)

   b. 600, 540, 486, 437.4, 393.66, …………(CHALLENGE: Find the SUM of the first ten terms, first 10,000 terms, the sum of an infinite number of terms)

6. (CA) Use your TI-84 calculator to establish a new connection between exponents and logarithms as you evaluate the following: {1,6}

(a) $2^3 = ?$ $\log_2 8 = ?$ (b) $3^5 = ?$ $\log_3 243 = ?$ (c) $9^{\frac{1}{2}} = ?$ $\log_9 3 = ?$ (d) $10^{3.5} = ?$ $\log_{10} 3162.28 = ?$ (e) $5^{-3} = ?$ $\log_5\left(\frac{1}{125}\right) = ?$ (e) $\left(\frac{1}{2}\right)^4 = ?$ $\log_{\frac{1}{2}}\left(\frac{1}{16}\right) = ?$

7. (CA) The population of the USA can be modeled by the equation $P(t) = 227e^{0.0093t}$, where P is population in millions and t is time in years since 1980. {6,11,20}

   a. What is the predicted population in 2015?
   b. What assumptions are being made in question Q(a)?
   c. When will the population reach 500 million?
   d. What is the annual growth rate?

8. (**CA**) Two equations in a linear-exponential system are given: $f(x) = \frac{3}{2}(x+2) - 5$ and $g(x) = 5 - 3\left(\frac{2}{3}\right)^x$.
Solve this system and hence solve the inequality $g(x) > f(x)$. {10}

Higher Level Questions for More Complex Concepts OR an EXTENSION of basic concepts involved with Exponential and Logarithmic Functions.

1. A trainee is hired by a computer manufacturing company. This trainee must learn how to test personal computers as they come off the assembly line. The learning curve (research this … what is a "learning curve"?) for an "average" new trainee is given by $N(t) = \dfrac{200}{4 + 21e^{-0.1t}}$.
   a. How many computers can the average trainee be expected to test after 3 days of training? After 6 days? Round to the nearest integer.
   b. How manty days will it take before the training can test 30 computers per day?
   c. Does *N* approach a limiting value as t increases without bound?

2. Working with the function $f(x) = \dfrac{8}{1 + 3e^{-0.5x}}$,
   a. Determine the value of $f(0)$.
   b. Determine the "end behaviour" as $x \to +\infty$ (say, "evaluate" $f(1000)$).
   c. Determine the "end behaviour" as $x \to -\infty$ (say, "evaluate" $f(-1000)$).
   d. Is $f(x) = \dfrac{8}{1 + 3e^{-0.5x}}$ an even/odd/neither function? Explain how you know.
   e. Graph the function $f(x) = \dfrac{8}{1 + 3e^{-0.5x}}$. This function is called a "logistic growth function" and can be used to model population growth. Explain why.