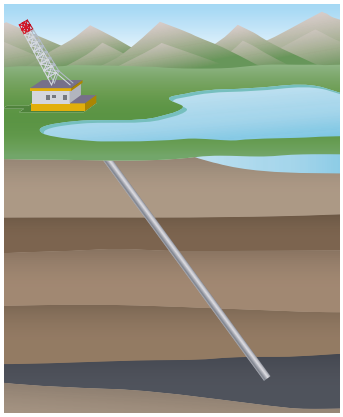
7.6 Solving Right Triangle Problems

GOAL

Use the primary trigonometric ratios to solve problems that involve right triangle models.



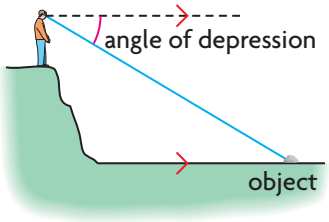
LEARN ABOUT the Math

Jackie works for an oil company. She needs to drill a well to an oil deposit. The deposit lies 2300 m below the bottom of a lake, which is 150 m deep. The well must be drilled at an angle from a site on land. The site is 1000 m away from a point directly above the deposit.

? At what angle to Earth's surface should Jackie drill the well?

angle of depression (angle of declination)

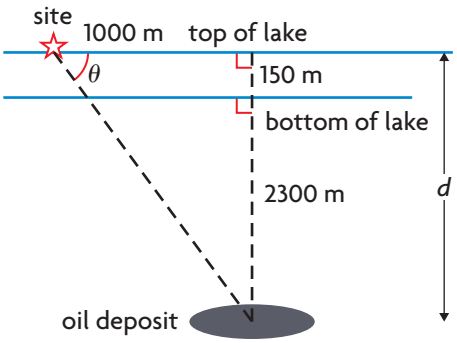
the angle between the horizontal and the line of sight when looking down at an object



EXAMPLE 1 **Solving a problem** using a right triangle model

Determine the angle at which the well should be drilled.

Jackie's Solution



I drew a diagram that shows where the lake, oil deposit, and drill site are located.

I added a line to show the angle of depression to the deposit.

I labelled the angle of depression θ.

$$d = 150 + 2300$$
$$= 2450$$

I calculated the length of the side that is opposite angle θ by adding the two given vertical distances.

$$\tan\theta = \frac{2450}{1000}$$
$$\tan\theta = 2.45$$
$$\theta = \tan^{-1}(2.45)$$
$$\theta \doteq 68°$$

To calculate θ, I used my calculator. Since I knew the opposite and adjacent sides to θ, I used the inverse tangent.

The well should be drilled at an angle of about 68°.

Reflecting

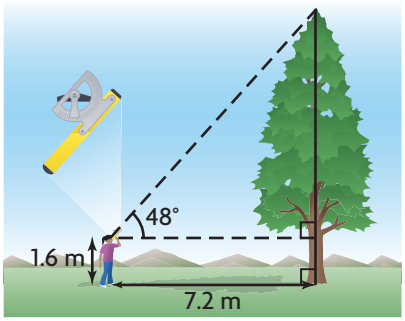
A. How does an angle of depression relate to an angle of elevation?

B. How could Jackie calculate the distance from the oil deposit to the drill site?

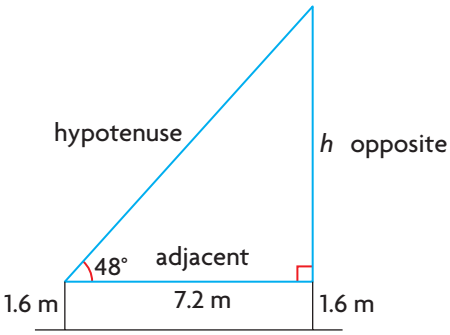
APPLY the Math

EXAMPLE 2 **Solving a problem** using a clinometer

Ayesha is a forester. She uses a clinometer (a device used to measure angles of elevation) to sight the top of a tree. She measures an angle of 48°. She is standing 7.2 m from the tree, and her eyes are 1.6 m above ground. How tall is the tree?



Joan's Solution



I drew a diagram to model the problem. The height is the length of the side that is opposite the 48° angle. I named the rest of the sides in the triangle relative to the 48° angle.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 48° = \frac{h}{7.2}$$

Because I knew the adjacent side, I used the tangent ratio. The tangent of 48° equals the opposite side, or height, divided by the adjacent side.

$$7.2\,(\tan 48°) = 7.2\left(\frac{h}{7.2}\right)$$

I multiplied both sides by 7.2 and evaluated.

$$7.2\,(\tan 48°) = h$$

$$8.00 \doteq h$$

$$\text{tree height} = 1.6 + 8.0$$

$$= 9.6$$

I added the distance from the ground to Ayesha's eyes to the height of the triangle to calculate the height of the tree.

The tree is about 9.6 m tall.

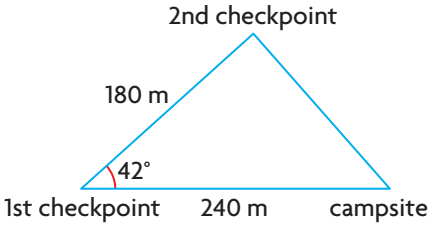
EXAMPLE 3 **Solving an area problem** when the height is unknown

A group of students are on an outdoor education trip. They leave their campsite and travel 240 m before reaching the first orienteering checkpoint. They turn, creating a 42° angle with their previous path, and travel another 180 m to get to the second checkpoint. They turn again and travel the shortest possible path back to their campsite. What area of the woods did their triangular route cover?

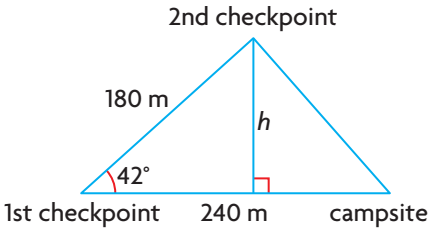
Safety *Connection*

A compass, map, first-aid kit, and signal device are important pieces of equipment when hiking in the woods.

Hugo's Solution



I created a diagram to represent the situation.



I had to determine the height of the triangle to calculate its area. I drew a line perpendicular to the 240 m side. I labelled this line as *h*.

$$\sin 42° = \frac{h}{180}$$
$$180(\sin 42°) = h$$
$$120.4 \doteq h$$

In the right triangle that contains the 42° angle, *h* is the side opposite this angle. I knew the hypotenuse, so I used the sine ratio to solve for *h*.

The height of the triangle is about 120.4 m.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(240)(120.4)$$
$$A = 14\,448$$

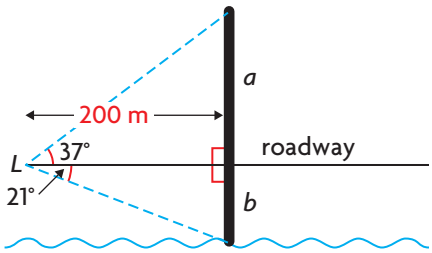
I calculated the area.

The area of the woods that the triangular route covered was about 14 448 m^2.

EXAMPLE 4 Solving a problem using two right triangles

Lyle stood on land, 200 m away from one of the towers on a bridge. He reasoned that he could calculate the height of the tower by measuring the angle to the top of the tower and the angle to its base at water level. He measured the angle of elevation to its top as 37° and the angle of depression to its base as 21°. Calculate the height of the tower from its base at water level, to the nearest metre.

Jenna's Solution



I started by drawing a diagram of the bridge and tower, and labelling the given angles. I split the height of the tower in two. I named the upper part of the tower (above roadway) *a* and the lower part of the tower (below roadway) *b*. This created two right triangles.

$$\tan 37° = \frac{a}{200}$$
$$200(\tan 37°) = a$$
$$150.7 \doteq a$$

I had to determine the side that is opposite the 37° angle in the top triangle. I knew the adjacent side, so I used the tangent ratio.

$$\tan 21° = \frac{b}{200}$$
$$200(\tan 21°) = b$$
$$76.8 \doteq b$$

I had to determine the side that is opposite the 21° angle in the bottom triangle. I knew the adjacent side, so I used the tangent ratio again.

$$\text{height} = a + b$$
$$= 150.7 + 76.8$$
$$= 227.5$$

I calculated the height of the tower by adding *a* and *b*.

The tower is about 228 m tall from its base at water level.

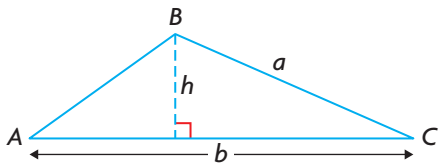
In Summary

Key Idea

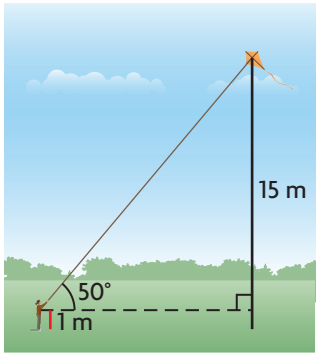
- If a problem involves calculating a side length or an angle measure, try to represent the problem with a model that includes right triangles. If possible, solve the right triangles using the primary trigonometric ratios.

Need to Know

- To calculate the area of a triangle, use the sine ratio to determine the height. For example, suppose that you know *a*, *b*, and $\angle C$ in the triangle at the right. To calculate the height, you can use $\sin C = \frac{h}{a}$, so $h = a(\sin C)$. Area of the triangle $= \frac{1}{2} \times b \times a(\sin C)$.

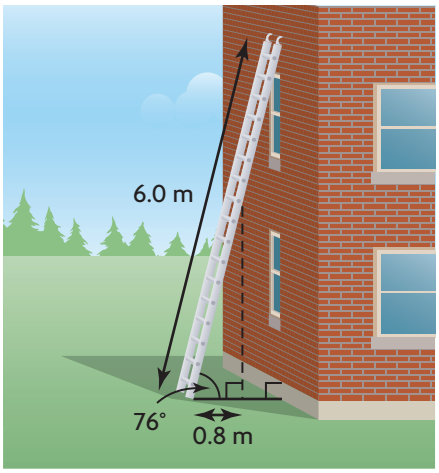


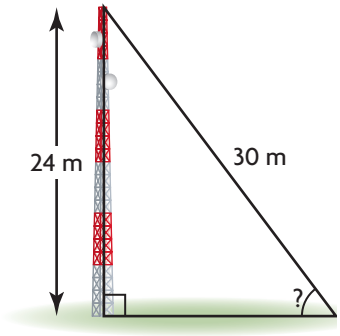
CHECK Your Understanding



1. Isabelle is flying a kite on a windy day. When the kite is 15 m above ground, it makes an angle of 50° with the horizontal. If Isabelle is holding the string 1 m above the ground, how much string has she released? Round your answer to the nearest metre.

2. Bill was climbing a 6.0 m ladder, which was placed against a wall at a 76° angle. He dropped one of his tools directly below the ladder. The tool landed 0.8 m from the base of the ladder. How far from the top of the ladder was Bill?





3. A guy wire is attached to a cellphone tower as shown at the left. The guy wire is 30 m long, and the cellphone tower is 24 m high. Determine the angle that is formed by the guy wire and the ground.

PRACTISING

4. A tree that is 9.5 m tall casts a shadow that is 3.8 m long.
K What is the angle of elevation of the Sun?

5. The rise of a rafter drops by 3 units for every 5 units of run. Determine the angle of depression of the rafter.

6. A building code states that a set of stairs cannot rise more than 72 cm for each 100 cm of run. What is the maximum angle at which the stairs can rise?

7. A contractor is laying a drainage pipe. For every 3.0 m of horizontal pipe, there must be a 2.5 cm drop in height. At what angle should the contractor lay the pipe? Round your answer to the nearest tenth of a degree.

Career *Connection*

Jobs in construction include designer, engineer, architect, project manager, carpenter, mason, electrician, plumber, and welder.

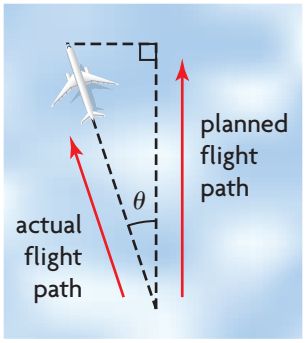
8. Firefighters dig a triangular trench around a forest fire to prevent the fire from spreading. Two of the trenches are 800 m long and 650 m long. The angle between them is 30°. Determine the area that is enclosed by these trenches.

9. A Mayan pyramid at Chichén-Itzá has stairs that rise about 64 cm for every 71 cm of run. At what angle do these stairs rise?

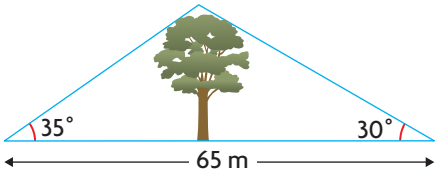
History *Connection*

Chichén-Itzá, in the Yucatan peninsula of Mexico, was part of the Mayan civilization. The pyramid called El Castillo, or the castle, is a square-based structure with four staircases and nine terraces.

10. After 1 h, an airplane has travelled 350 km. Strong winds, however, have caused the plane to be 48 km west of its planned flight path. By how many degrees is the airplane off its planned flight path?



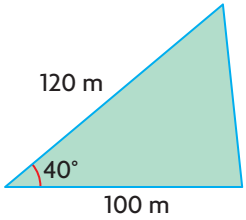
11. A Angles were measured from two points on opposite sides of a tree, as shown. How tall is the tree?



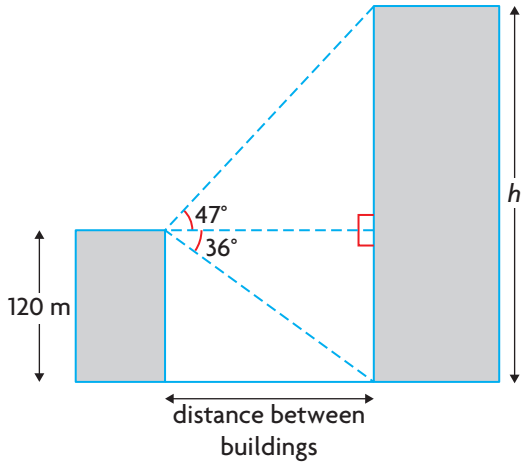
12. Determine the angle between the line $y = \frac{3}{2}x + 4$ and the x-axis.

13. T A bridge is going to be built across a river. To determine the width of the river, a surveyor on one bank sights the top of a pole, which is 3 m high, on the opposite bank. His optical device is mounted 1.2 m above the ground. The angle of elevation to the top of the pole is 8.5°. How wide is the river?

14. C Élise drew a diagram of her triangular yard. She wants to cover her yard with sod. Explain how you could calculate the cost, if sod costs \$1.50/m^2.



15. A video camera is mounted on top of a building that is 120 m tall. The angle of depression from the camera to the base of another building is 36°. The angle of elevation from the camera to the top of the same building is 47°.



a) How far apart are the two buildings? Round your answer to the nearest metre.

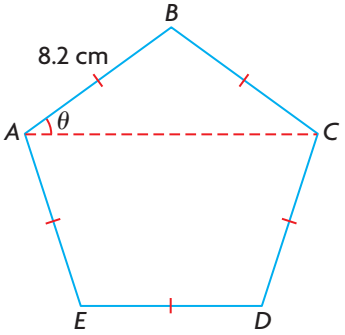
b) How tall is the building viewed by the camera? Round your answer to the nearest metre.

16. An isosceles triangle has a height of 12.5 m (measured from the unequal side) and two equal angles that measure 55°. Determine the area of the triangle.

17. To photograph a rocket stage separating, Lucien mounts his camera on a tripod. The tripod can be set to the angle at which the stage will separate. This is where Lucien needs to aim his lens. He begins by aiming his camera at the launch pad, which is 1500 m away. The stage will separate at 20 000 m. At what angle should Lucien set the tripod?

18. Explain the steps you would use to solve a problem that involves a right triangle model and the use of trigonometry.

Extending



19. Each side length of regular pentagon *ABCDE* is 8.2 cm.

a) Calculate the measure of θ to the nearest degree.

b) Calculate the length of diagonal *AC* to the nearest tenth of a centimetre.

20. Determine the acute angle at which $y = 2x - 1$ and $y = 0.5x + 2$ intersect.