(A) Lesson Context

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease? • What is a parabola and what key features do they have that makes them useful in modeling applications • How do I use graphs, data tables and algebra to analyze quadratic equations? • What algebraic strategies come into play for this new type of data relationship?

(B) Lesson Objectives:

a. Review the algebraic skills and strategies for solving equations and of factoring
b. Understand what solving a quadratic equation in the form of $ax^2 + bx + c = 0$ means in terms of graphs and functions
c. Use the skills of factoring to solve quadratic equations

(C) Opening Exercise: Skills Review

(CI) Each of the following quadratic equations DO NOT factor, so convert the equations into vertex form and solve by using the strategy of inversing operations (NOTE: you may use a calculator to determine square roots ……)

(i) $f(x) = x^2 + 6x - 4$ (ii) $g(x) = x^2 - 5x + 3$ (iii) $h(x) = 2x^2 + 4x - 1$ (iv) $y = x^2 + bx + c$

(D) *Opening Exercises: Applications of Quadratic Equations*

1. (CI) A model rocket is shot into the air and its path is approximated by $h(t) = -5t^2 + 30t$, where h is the height of the rocket above the ground in meters and t is the elapsed time in seconds.

   a. When will the rocket hit the ground?
   b. What is the maximum height of the rocket and at what time does it reach this height?

2. (CI) A company that manufactures snowboards uses the relation $P(x) = 162x - 81x^2$ to model its profit. In this model, x represents the number of snowboards in thousands and P represents the profit in thousands of dollars.

   a. What does the point (1.5, 60.75) mean in this context?
   b. What is the maximum profit the company can earn and how many snowboards must they sell to reach this maximum profit?
   c. How many snowboards must they see in order to break even?

3. (CI) A software company models the profit of its latest game using $P(x) = -2x^2 + 28x - 90$, where x represents the number of games it produces in hundreds of thousands and P represents the profit in millions of dollars.

   a. What does the point (4, -10) mean in this context?
   b. What is the maximum profit the company can earn and how many games must they produce to reach this maximum profit?
   c. How many games must they produce in order to break even?

(E) Solving Quadratic Equations by Factoring

1) $x^2 - 3x = 0$ 2) $z^2 - 5z = 0$ 3) $x^2 - 8x = 0$

4) $x^2 - 121 = 0$ 5) $z^2 - 9z = 0$ 6) $m^2 - 144 = 0$

7) $3x^2 - 12 = 0$ 8) $d^2 - 2 = 0$ 9) $s^2 - 36s = 0$

10) $2x^2 - 5x + 2 = 0$ 11) $3x^2 - 10x + 3 = 0$ 12) $3x^2 - 8x + 4 = 0$

13) $5x^2 + 11x = -2$ 14) $y^2 = 8y + 20$ 15) $x^2 = 9x - 20$

16) $x^2 = 30 + x$ 17) $2x^2 - x = 15$ 18) $x^2 + 3x - 4 = 50$

19) $2x^2 + 7 = 5 - 5x$ 20) $x(x - 2) = 35$ 21) $y(y - 3) = 4$

22) $\frac{x+2}{2} = \frac{12}{x}$ 23) $\frac{y+3}{3} = \frac{6}{y}$ 24) $\frac{x}{3} = \frac{12}{x}$

25) $10x^2 - 5x + 11 = 9x^2 + x + 83$ 26) $4x^2 + 3x - 12 = 6x^2 - 7x - 60$

(F) Application Problems

1. The area of a rectangle is given by $A = x^2 + 18x + 72$.

   a. Use factoring to find an expression for the dimensions of the rectangle.
   b. If the area of the rectangle is 7 square feet, what are the possible values of x?
   c. What are the dimensions of the rectangle?

2. Recall the area of a circle is given by $A = \pi r^2$, where r is the radius of the circle.
   a. If a particular circle is given by $A = \pi(x^2 - 20x + 100)$, find an expression for the radius of the circle.
   b. If the area of the circle is 16π square feet, what is the value of x?

3. The product of two consecutive odd integers is 1 less than four times their sum. Find the two integers. Hint: There will be two sets of solutions.

4. The hypotenuse of a right triangle is 6 more than the shorter leg. The longer leg is three more than the shorter leg. Find the length of the shorter leg. Hint: Draw a right triangle and apply the Pythagorean Theorem.

5. Extension #1 ➔ Sketch a graph of a quadratic function that would not have any real number solutions and explain why there would not be any real number solutions.

6. Extension #2 ➔ Two cars leave an intersection. One car travels north and the other car travels east. When the car traveling north had gone 24 miles, the distance between the cars was four miles more than three times the distance traveled by the car heading east. Find the distance between the cars at that time.

(G) Solving (by Factoring) Quadratic Equations ➔ CI Application Problems

18. **Thinking, Inquiry, Problem Solving:** Soundz Inc. makes CD players. Last year, accountants modelled the company's profit by $P = -5x^2 + 60x - 135$. Over the course of the year, in an effort to become more efficient, Soundz Inc. restructured its operation, eliminating some employees and reducing costs. This year, accountants are using $P = -7x^2 + 70x - 63$ to project the company's profit. In both models, P is the profit in hundreds of thousands of dollars and x is the number of CD players made, in hundreds of thousands. Was Soundz Inc.'s restructuring effective? Justify your answer.