(A) Lesson Context

BIG PICTURE of this UNIT:	• How do we analyze and then work with a data set that shows both increase and decrease • What is a parabola and what key features do they have that makes them useful in modeling applications • How can I use graphs and equations of quadratic relations to make predictions from data sets & their models • How do I use graphs, data tables and algebra to analyze quadratic equations?

(B) Lesson Objectives:

a. Review and preview fundamental algebra skills
b. Prepare scatterplots of data and predict quadratic equations
c. Use Quad Reg to get the regression equation for data sets

(C) Algebra of Polynomials:

1. For the following quadratic equations,

   (i) state the location of the vertex and the "direction of opening"
   (ii) expand and simplify to express the equation in standard form:

   (a) $y = (x - 6)^2 + 5$ (b) $y = 5 + 2(x + 3)^2$ (c) $y = ½(2x - 5)^2$ (d) $y = -4(2x + 3)^2 - 3$

2. For the following equations, expand and simplify to express the equation in standard form.

   (a) $y = (2x + 3)(x + 6)$ (b) $y = -2(2x + 5)(x + 7)$ (c) $y = 5(x + 2)(3x - 2)$ (d) $y = -½(4x - 3)(5 - x)$

3. The following quadratic equations have been given to you in standard form (as I already have expanded and simplified them.) Now work **backwards** to find out what expressions I expanded in order to get these equations

   i.e. the standard equation of $y = x^2 + 2x - 8$ came from the expansion of $y = (x - 2)(x + 4)$

   (a) $y = x^2 - x - 6$ (b) $y = x^2 + 6x - 16$ (c) $y = 2x^2 + 5x - 3$ (d) $y = 4x^2 - 4x - 15$

(D) Investigating Quadratic Relations – Geometry Problems – Maximizing Area

(a) a rectangular field is to be fenced using 40 meters of fencing. Build different "fields" that could be constructed, given the fact that all 40 meters must be used. For each field, record the length, the width, and the area that results. Finally, what are the dimensions of the field that maximize the area?

W							
L							
A							

(b) Now for some variations on the rectangular field problem. The field backs onto a river, so now build the different "fields" that could be constructed, given the fact that all 40 meters must be used. For each field, record the length, the width, and the area that results. Finally, what are the dimensions of the field that maximize the area?

W							
L							
A							

(c) Now for some variations on the rectangular field problem. The field backs onto a river and will be divided into 2 adjacent fields, so now build the different "fields" that could be constructed, given the fact that all 40 meters must be used. For each field, record the length, the width, and the area that results. Finally, what are the dimensions of the field that maximize the area?

W							
L							
A							

(d) Finally, let's graph and try to write equations that model the relationship between the area of the field and its width. Now write an equation that models the relationship between the area of the field and its length. Are the equations the same or are the different? How do the equation(s) allow us to answer the original question ➔ what dimensions of the field will maximize the area enclosed by the fencing.

Enter data here ➔
https://docs.google.com/spreadsheets/d/1b7sgWxBZZ8c1XzVNDmk8cAhASy68b2zqUxYfgoQp7tI/edit?usp=sharing

(E) ***Investigation – Investigating the Graphs of Quadratic Functions & Factored Form***

QUESTION ➔ All of the quadratics you will graph are presented in the form of $y = \boldsymbol{a}(x - \boldsymbol{s})(x - \boldsymbol{t})$. How do the values of ***a***,***s***,***t*** affect the graph?

To record your groups findings & ideas ➔ open a google doc & share it with your group members & with me

1. Use a graphing calculator (or use www.desmos.com) to graph $y = \boldsymbol{a}(x - 2)(x + 6)$ when $\boldsymbol{a} = 3$. Describe what happens to the graph as you change the value of ***a*** to 2, 1, ½, ¼ , 0, -1, -2, -3. Include sketches. Where is the axis of symmetry in each parabola?

2. Graph $y = 2(x - \boldsymbol{s})(x + 5)$ when $\boldsymbol{s} = 3$. Describe what happens to the graph as you change the value of ***s*** to 2, 1, 0, -1, -2, -3. Include sketches.

3. Find the axis of symmetry of each parabola you investigated in Q2.

4. Which of the quantities ***a***, ***s***, or ***t*** affects whether the graph has a maximum or a minimum value? How can you PREDICT where a parabola has a maximum or minimum?

5. Which of the quantities ***a***, ***s***, or ***t*** affects where the graph has a zeroes? How can you PREDICT where a parabola has its zeroes?

(F) Consolidation of Investigations ➔ Key Points

a. Equations in the form of y = $\boldsymbol{a}$(x – $\boldsymbol{s}$)(x – $\boldsymbol{t}$) are________________________, provided that .

b. The equation written the form y = $\boldsymbol{a}$(x – $\boldsymbol{s}$)(x – $\boldsymbol{t}$) is said to be in _________________________.

c. If a > 0, the parabola opens ________________________and has __________________________.

d. If a < 0, the parabola opens ________________________and has __________________________.

e. The zeroes of the quadratic can be determined by setting ___________and solving______________. The zeroes are then located ___________________________________.

f. If the zeroes are known, then the axis of symmetry can be found ➔_________________________.

g. Once the axis of symmetry is known, the optimal value can be found ➔______________________.

h. The value of $\boldsymbol{a}$ can be determined IF_______________________________. All known values are substituted into y = $\boldsymbol{a}$(x – $\boldsymbol{s}$)(x – $\boldsymbol{t}$) and then solve for $\boldsymbol{a}$.

(G) Examples

a. Ex 1 ➔ For the quadratic relation y = (x + 3)(x – 4), determine:

- ***i.*** The direction of opening.
- ***ii.*** The zeroes
- ***iii.*** The optimal point.
- ***iv.*** The y-intercept.
- ***v.*** Sketch the parabola.

b. Ex 2 ➔ The zeroes of a parabola are -3 and 5. The graph crosses the y-axis at -75. Determine:

- ***i.*** if the relation have a maximum or minimum value?
- ***ii.*** the equation of the quadratic relation.
- ***iii.*** the co-ordinates of the vertex.
- ***iv.*** Sketch the parabola.

5. For each relation, state

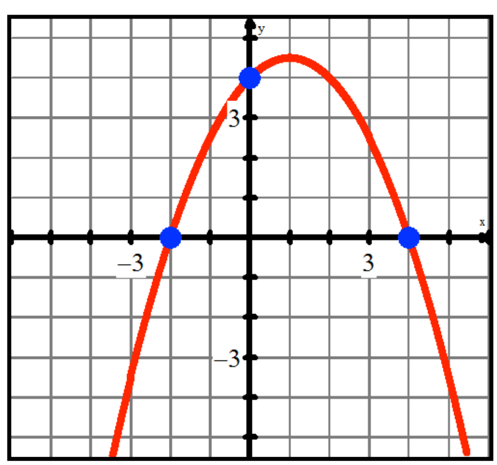
i. the x-intercepts
ii. the equation of the axis of symmetry
iii. the coordinates of the vertex

(a) $y = (x + 4)(x + 2)$
(b) $y = (x + 5)(2 - x)$
(c) $y = (4 + x)(1 + x)$
(d) $y = (1 - x)(3 + x)$
(e) $y = (x - 3)(2 - x)$
(f) $y = (x + 1)(x - 4)$
(g) $y = 3(x + 1)(x - 3)$
(h) $y = -2(x + 3)(x - 3)$

7. Sketch a graph for each relation. Do not make a table of values or use graphing technology.

(a) $y = (x + 3)(x + 5)$
(b) $y = (x - 3)(x - 5)$
(c) $y = (x - 6)(x - 2)$
(d) $y = -(x - 1)(x - 2)$
(e) $y = 3(x - 5)(x + 1)$
(f) $y = -2(x + 2)(x + 1)$
(g) $y = \frac{1}{2}(x - 4)(x - 2)$
(h) $y = -2(3 - x)(5 - x)$
(i) $y = 10(x - 1)(x + 6)$

Find the equation of this parabola ➔



(H) **Application/Context Problems**

a. Ex 1 ➔ Mr. S throws a ball upward from the roof of the building that is 25m tall. The ball reaches a height of 45m above the ground after 2s and hits the ground 5s after being thrown.

- ***i.*** Draw an accurate graph of the height of ball and the time in flight.
- ***ii.*** What are the zeroes of the relation?
- ***iii.*** What are the co-ordinates of the vertex?
- ***iv.*** Determine an equation that models this situation.
- ***v.*** What is the meaning of each zero?

b. Ex 2 ➔ a company called SAMSOONG introduces a new cellphone and its PROFITS are modelled by the equation $P(m) = -5m^2 + 80m - 100$ where m is time in months and P(m) is the profit in millions of dollars. The cellphone is sold for a period of 2 years.

- ***i.*** Graph the profit function on your TI-84.
- ***ii.*** Calculate the zeroes of the quadratic and interpret what they mean.
- ***iii.*** Write the equation in factored form, given your work in (ii).
- ***iv.*** Calculate the co-ordinates of the vertex and interpret.
- ***v.*** Evaluate P(5) and interpret.
- ***vi.*** Solve $P(m) = -25$ and interpret
- ***vii.*** Solve $P(m) < 0$ and interpret
- ***viii.*** For what values of m are the profits DECREASING? Explain how you determined your answer.