A. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

B. Lesson Objectives

i. Review Basics of Exponential Algebra
ii. Study applications of exponential functions

PART 2 – Skills PRACTICE

C. Working with Rational Exponents and Indices – Simplify the following expressions

1) $(n^4)^{\frac{3}{2}}$	2) $(27p^6)^{\frac{5}{3}}$
3) $(25b^6)^{-1.5}$	4) $(64m^4)^{\frac{3}{2}}$
5) $(a^8)^{\frac{3}{2}}$	6) $(9r^4)^{0.5}$
7) $(81x^{12})^{1.25}$	8) $(216r^9)^{\frac{1}{3}}$
15) $\dfrac{3x^{-\frac{1}{2}} \cdot 3x^{\frac{1}{2}}y^{-\frac{1}{3}}}{3y^{-\frac{7}{4}}}$	16) $\dfrac{3y^{\frac{1}{4}}}{4x^{-\frac{2}{3}}y^{\frac{3}{2}} \cdot 3y^{\frac{1}{2}}}$
21) $\dfrac{\left(x^{-\frac{1}{2}}y^2\right)^{-\frac{5}{4}}}{x^2y^{\frac{1}{2}}}$	22) $\dfrac{\left(x^{-\frac{1}{2}}y^4\right)^{\frac{1}{4}}}{x^{\frac{2}{3}}y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}}y^{\frac{1}{2}}}$

https://cdn.kutasoftware.com/Worksheets/Alg2/Simplifying%20Rational%20Exponents.pdf

D. Working with Exponent Laws

15) $ba^4 \cdot (2ba^4)^{-3}$	16) $(2x^0y^2)^{-3} \cdot 2yx^3$
17) $\dfrac{2k^3 \cdot k^2}{k^{-3}}$	18) $\dfrac{(x^{-3})^4 x^4}{2x^{-3}}$
19) $\dfrac{(2x)^{-4}}{x^{-1} \cdot x}$	20) $\dfrac{(2x^3z^2)^3}{x^3y^4z^2 \cdot x^{-4}z^3}$
21) $\dfrac{(2pm^{-1}q^0)^{-4} \cdot 2m^{-1}p^3}{2pq^2}$	22) $\dfrac{(2hj^2k^{-2} \cdot h^4j^{-1}k^4)^0}{2h^{-3}j^{-4}k^{-2}}$

https://cdn.kutasoftware.com/Worksheets/Alg1/Properties%20of%20Exponents.pdf

E. Working with Exponential Equations

13) $4^{-2x} \cdot 4^x = 64$	14) $6^{-2x} \cdot 6^{-x} = \dfrac{1}{216}$
15) $2^x \cdot \dfrac{1}{32} = 32$	16) $2^{-3p} \cdot 2^{2p} = 2^{2p}$
17) $64 \cdot 16^{-3x} = 16^{3x-2}$	18) $\dfrac{81^{3n+2}}{243^{-n}} = 3^4$
19) $81 \cdot 9^{-2b-2} = 27$	20) $9^{-3x} \cdot 9^x = 27$

https://cdn.kutasoftware.com/Worksheets/Alg2/Exponential%20Equations%20Not%20Requiring%20Logarithms.pdf

Evaluate each expression.

21) $\log_4 64$

22) $\log_6 216$

23) $\log_4 16$

24) $\log_3 \frac{1}{243}$

25) $\log_5 125$

26) $\log_2 4$

27) $\log_{343} 7$

28) $\log_2 16$

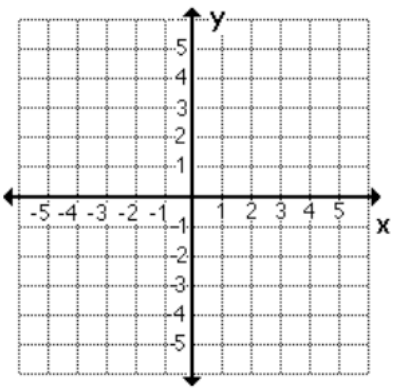
29) $\log_{64} 4$

30) $\log_6 \frac{1}{216}$

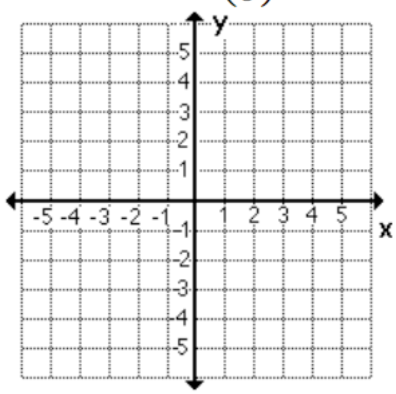
https://cdn.kutasoftware.com/Worksheets/Alg2/Meaning%20of%20Logarithms.pdf

F. Working with Graphs of Exponential Functions

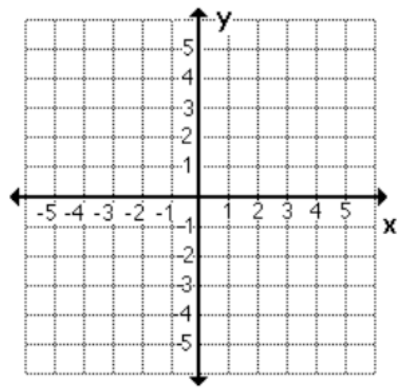
10. $f_{(x)} = 3^{x-2} + 2$



11. $f_{(x)} = \left(\frac{2}{3}\right)^{-x}$

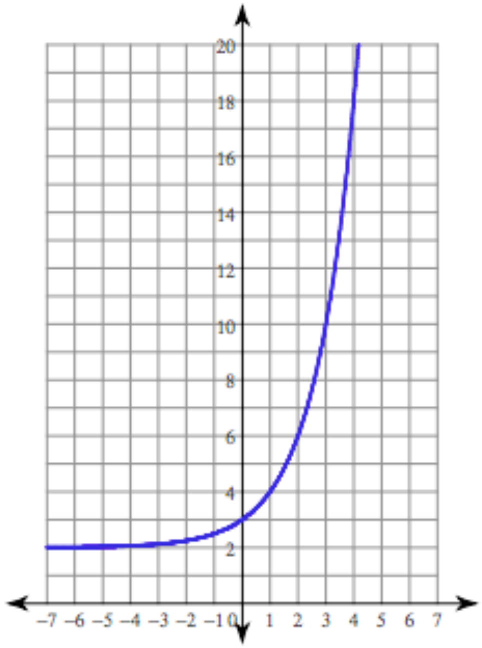


12. $f_{(x)} = 2^{x+2} + 1$

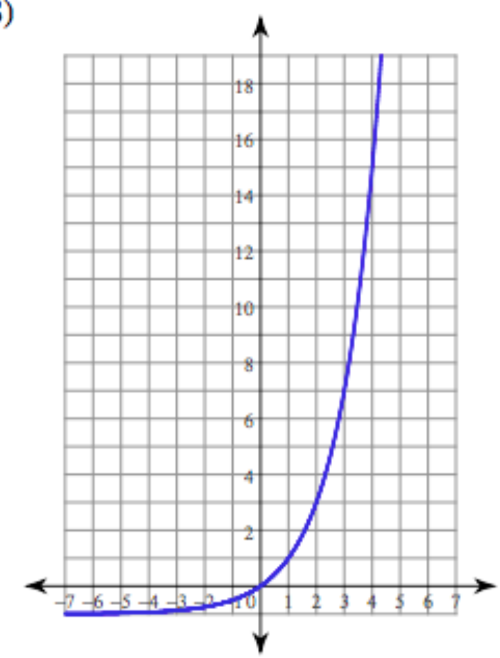


Write an equation for each graph.

7)



8)



PART 1 – Concept Investigations

G. Word Problems – Rate Changes

a. A population of 100 mice will quadruple every year.

i. What will be the population after 4 years?
ii. How long will it take to get 10,00 mice?
iii. Determine the WEEKLY growth rate for the mice.
iv. What assumptions are being made for this growth model? How reasonable is this assumption?
v. Now, graph the model $P(t) = \dfrac{10{,}000}{1+4^{-0.1t}}$. How long does it take for the mouse population to reach 10,000? Why? Explain why is this model more realistic for the mouse population?

b. A colony of 100,000 ants is infected by a virus and its monthly population is modeled by the following function: $P(m) = 100000(0.88)^m$.

i. How many ants will be in the colony after 10 months?
ii. How long will it take to get 25,000 ants in the colony?
iii. Determine the half-life of the ant population in this model for the decrease of the ant population. Write the equation using the half-life "formula."
iv. Determine the DAILY death rate for the ant colony.

c. A baby weighing 7 pounds at birth may increase in weight every month according to the function $W(m) = 7(1.41)^m$.

i. How much will the baby weigh after 1 year?
ii. When will the baby weigh 18 pounds?
iii. When will the baby weigh 180 pounds? Is this reasonable?
iv. Determine the approximate DAILY rate of growth for this infant.
v. Mr. S. suggests the following piecewise defined model for the weight-age relationship (where y is the weight and x is time in years: $y = \begin{cases} 7(1.41)^x & 0 \le x \le 5 \\ 10(x-5)+39 & 5 \le x \le 17 \\ 200 - 40(0.75)^{x-17} & x > 17 \end{cases}$. Graph this function and explain why it might be a more reasonable model for the weight-age relationship? Now, when does the child expect to weigh 180 pounds?

Extending

19. Liz decides to save money to buy an electric car. She invests \$500 every 6 months at 6.8%/a compounded semi-annually. What total amount of money will she have at the end of the 10th year?

In each of problems 3 and 4, there are three functions defined by a table. In each case one of the functions is linear, one of the functions is exponential, and one of the functions is neither. In each case, *(a)* identify the linear and exponential functions, *(b)* find a formula for the linear function, *(c)* find a formula for the exponential function, and *(d)* try to guess a formula for the third function.

3.

x	$f(x)$	$g(x)$	$h(x)$
−2	4	0	1/9
0	0	1	1/3
2	4	2	1
4	16	3	3
6	36	4	9

4.

x	$f(x)$	$g(x)$	$h(x)$
0	100.00	36.25	0
3	90.00	34.20	27
6	81.00	32.15	216
9	72.90	30.10	729
12	65.61	28.05	1728

For extra practice, complete the three challenge questions from this worksheet:

Extension Questions: QUESTION #1

Mr Smith's wife has just learned that she is pregnant! Mr. Smith wants to know when his new baby will arrive and decides to do some research. On the Internet, he finds the following article:

Hormone Levels for Pregnant Women

When a woman becomes pregnant, the hormone HCG (human chlorionic gonadotropin) is produced in order to enable the baby to develop.

During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. Commonly, the HCG levels are measured two days apart to look for this rate of growth.

A woman who is not pregnant will often have an HCG level of between 0 and 5 mIU (milli-international units) per ml (milliliter).

Then Smith remembered that his wife was tested for HCG during her last two doctor visits.

1. On March 21, her HCG level was 200 mIU/ml, while two days later, her HCG level was 392 mIU/ml.. Assuming that the model for HCG levels is of the form $y = ab^x$, what equation models the growth of HCG for his wife's pregnancy?

2. Assume her HCG level was 5 mIU on the day of implantation. How many days after implantation was his wife's first doctor visit? March 21? What day did the baby most likely become implanted?

3. Smith also learned that a baby is born approximately 37 weeks after implantation. What day can Smith expect to become a father?

QUESTION #2: <u>*Doses of Medicine*</u>

Medicine in the body decays in an exponential way. Mr. Smith is taking some medication. On Monday Mr. Smith took the pills. On Tuesday he had 15 mg of medicine left in his body. On Friday he had 6.328125 mg left in his body.

1. Create an exponential equation modeling this situation. (Remember… starting should be on Monday)

2. When the amount of medicine in Mr. Smith's body drops below 4 mg, he needs to take another pill. When does Mr. Smith need to take more medicine?

GIVEN: the formula for working with compound interest ➔ $FV = PV\left(1+\frac{i}{n}\right)^{nt}$, determine the value after 1 year of a $1 investment invested at 100% pa under the following compounding conditions:

Compounding condition	**Value of the money**
(a) 100% pa compounded annually	
(b) 100% pa compounded semi-annually	
(c) 100% pa compounded quarterly	
(d) 100% pa compounded monthly	
(e) 100% pa compounded daily	
(f) 100% pa compounded hourly	
(g) 100% pa compounded every minute	
(h) 100% pa compounded every second	
(i) 100% pa compounded n times per year	

a. FINAL QUESTIONS? ➔ BY WHAT **RATIO** HAS YOUR MONEY INCREASED IN VALUE?

b. FINAL QUESTIONS? ➔ WHAT DOES THE IDEA OF **COMPOUNDING CONTINUOUSLY MEAN**?

Given the expression $\left(1+\frac{1}{x}\right)^{x}$, prepare a table of values wherein x is: {1,2,3,4,5,10,15,20,50,75,100,250). Now predict the value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$.