A. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

B. Lesson Objectives

a. Use algebraic strategies to solve Exponential equations
b. Use multiple representations to verify algebraic solutions
c. Apply Exponential Equations to real world applications

PART 1 – Concept Investigations

C. Intro to Algebraic Strategies – Solving Exponential Equations

(a) Use ALGEBRAIC METHODS to solve and verify these equations. Finally, use your TI-84 to graphically verify.

(a) Solve and verify $2^{3-x} = 2^4$	(b) Solve and verify $2^{x-3} = 2^{3x+1}$
(c) Solve and verify $2^{2x+3} = 16$	(d) Solve and verify $8^x = 16^{x-1}$

(b) Examples of Equations to Solve

(a) $2^{1-2x} = 8$	(b) $4^{1+x} = 2$
(c) $3^{x+2} = \frac{1}{9}$	(d) $4^{2-x} = 5$
(e) $2^{3-2x} = 2^x$	(f) $4^{x-1} = 2^x$
(g) $\left(\frac{1}{4}\right)^{2x+1} = \left(\frac{1}{8}\right)^{3-x}$	(h) $3^{2x-2} = 2^x$

PART 2 – Skills PRACTICE

D. Word Problems: Using Exponential Equations – Solving algebraically in context: $y = a(1 + r)^x$

Solving for y:

1. You deposit $1600 in a bank account. Find the balance after 3 years if the account pays 2.5% annual interest compounded quarterly.
2. The population of HS students at CAC can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. If there were 350 students present in 2013, how many HS students would be predicted to be at CAC in 2020?

Solving for a:

3. In 8 years, you want the money you invest to reach $10,000. The account pays 8% annual interest compound monthly. How much money do you need to invest?
4. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. There are currently 320 HS students at CAC. How many where present in 2000?

Solving for r:

5. After investing $2000 for 15 years, you now have $8,000. What interest rate does the investment pay annually?
6. The population of HS students at CAC can be modeled with an exponential function. If there were 370 students present in 2011 and 315 students in 2014, what is the annual rate of decrease of student population in HS at CAC?
7. The value of land in New Cairo grows exponentially. Five years ago, 10 hectares of land cost 0.75 million LE and today, the same 10 hectares cost 2.5 million LE. Determine the annual rate of increase of the land.

Solving for t:

8. You buy a new computer for $2100. The computer decreases in value by 50% annually. When will the computer be worth $600?
9. The population of HS students at CAC since the year 2000 can be modeled with an exponential function. The number of students continues to decline at an annual rate of 11%. How long would it take for the student population to decline from 350 students to 250 students?
10. The value of land in New Cairo grows exponentially. Today 10 hectares of land cost 2.5 million LE and the value of the land is increasing at an annual rate of 17.5%. How long will it take for the land value to be 4.0 million LE?

E. Doubling Formula

Answer the following questions that deal with the doubling concept. Recall that the formula $y = ab^x$ which can now be rewritten as $y = a(2)^{\frac{t}{D}}$. In these two formulas, recall what the variables really mean:

1. A dish has 212 bacteria in it. The population of bacteria will double every 2 days. How many bacteria will be present in . . . a) 8 days b) 11 days c) 4 hours d) 2 months

2. An experiment starts off with *X* bacteria. This population of bacteria will double every 7 days and grows to 11,888 in 32 days. How many bacteria where present at the start of the experiment?

3. A bacteria culture grows according to the formula: $y = 12000(2)^{\frac{t}{4}}$ where t is in hours. How many bacteria are present:
   (a) at the beginning of the experiment?
   (b) after 12 hours?
   (c) after 19 days?
   (d) What is the doubling time of the bacteria?

4. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48 000 bacteria present. What is the length of the doubling period?

5. Mr S. makes an initial investment of $15,000. This initial investment will double every 9 years. What is the value of this investment in . . . a) 20 years b) 6 years c) What is the yearly rate of increase of this investment?

F. Half Life formula

Answer the following questions that deal with the doubling concept. Recall that the formula $y = ab^x$ which can now be rewritten as $y = a\left(\frac{1}{2}\right)^{\frac{t}{H}}$. In these two formulas, recall what the variables really mean:

6. Iodine-131 is a radioactive isotope of iodine that has a half-life of 8 days. A science lab initially has 200 grams of iodine-131. How much iodine-131 will be present in . . .
   a) 8 days b) 20 days c) 1 year d) 2 months

7. A medical experiment starts off with *X* grams of a radioactive chemical called Mathematus. This chemical will decay in half every 15 seconds and in the course of the experiment, will decay to 9.765 g in 2 minutes. How much Mathematus was present at the start of the experiment?

8. A chemical decays according to the formula: $y = 12000\left(\frac{1}{2}\right)^{\frac{t}{25}}$ where t is in time in hours and y is amount of chemical left, measured in grams. What amount of chemical is present:

(a) at the beginning of the experiment?
(b) after 100 hours?
(c) after 19 days?
(d) What is the half-life of the chemical?

9. A block of dry ice is losing its mass at a rate of 12.5% per hour. At 1 PM it weighed 50 pounds. What was its weight at 5 PM? What was the approximate half-life of the block of dry ice under these conditions?

G. <u>Systems Investigation</u>

My brother works as an electrician and runs his own company. In the first year of running his business, he earned total revenues of $250,000 and he now estimates that his annual revenue has been increasing at a rate of 30% of the previous year's revenues. He also realizes that his business has expenses, which he estimated at $100,000 for his first year of running his business. However his expenses have been increasing at a constant, fixed amount of $55,000 every year. You will analyze the profitability of his business using appropriate mathematical modeling.

(a) Write an equation for his company's REVENUES. Graph this equation on your TI-84. (Window settings x $\Rightarrow$ 0-25 and y $\Rightarrow$ 0 – 1,000,000)

(b) Write an equation for his company's EXPENSES. Graph this equation on the same axes as (i).

(c) If you know a company's revenues and expenses, how do you determine its PROFITS?

(d) Write an equation that will model the company's PROFITS.

(e) What is the company's profitability in the fifth year of operation?

(f) What is the company's profitability in the 7th year of operation?

(g) What do the intersection points represent?

(h) What ASSUMPTION are you making as you analyze my brother's company's profitability?