IM2 PS 4.7: Modeling with Exponential Functions – More work with Compound Interest	Unit 4 – Exponential Functions

A. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

B. Lesson Objectives

a. Use algebraic strategies to solve Exponential equations
b. Introduce and work with compounding interest

PART 1 – Concept Investigations

C. Algebraic Strategies - Solving for the variable & Reviewing Skills with Exponent Laws

$y = 3^3$

$y = 17^4$

$27 = x \cdot 3^2$

$500 = x \cdot 4^4$

$27 = x^3$

$76 = x^9$

$27 = 3^x$

$56 = 5^x$

Simplify: $\dfrac{(6x^3y^{-4})^{-2}}{(3x^2y^5)^{-3}}$

Simplify: $\dfrac{(-3x^{-4}y)^{-4}}{(5x^{-2}y^3)^0}$

Simplify: $\left(\dfrac{5x^{-4}y^2}{3x^{-1}y^{-3}}\right)^{-2}$

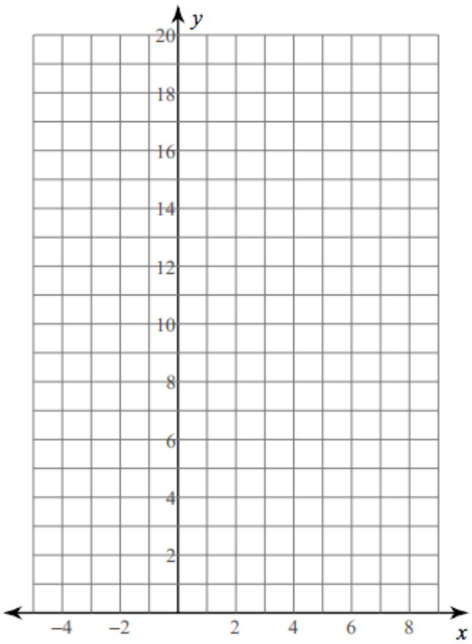
Simplify: $\dfrac{(4x^3y^{-2})^2}{(2x^2y^{-5})^0(6xy^{-3})^3}$

Simplify: $\left(\dfrac{7x^3y}{2x^{-5}y^2}\right)^0 \cdot (4x^{-3}y^2)^{-1}$

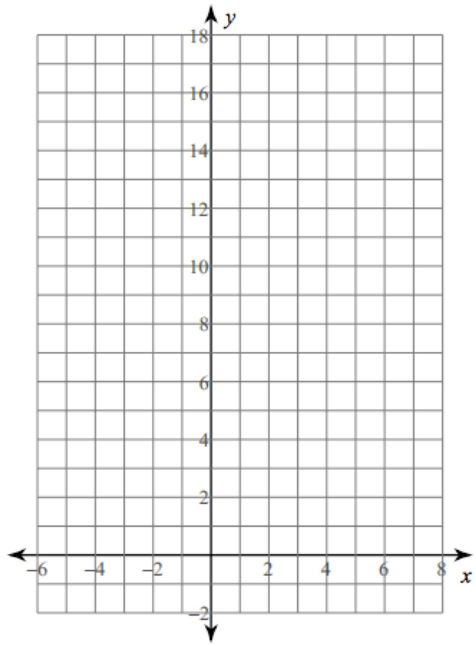
Simplify: $\dfrac{(8x^3y^{-4})^{-2}}{(-4x^{-1}y)^{-3}(2x^5y^{-3})^{-2}}$

D. Practicing Skills with Graphs of Exponential Functions

5) $y = 3 \cdot 2^{x-2} + 2$

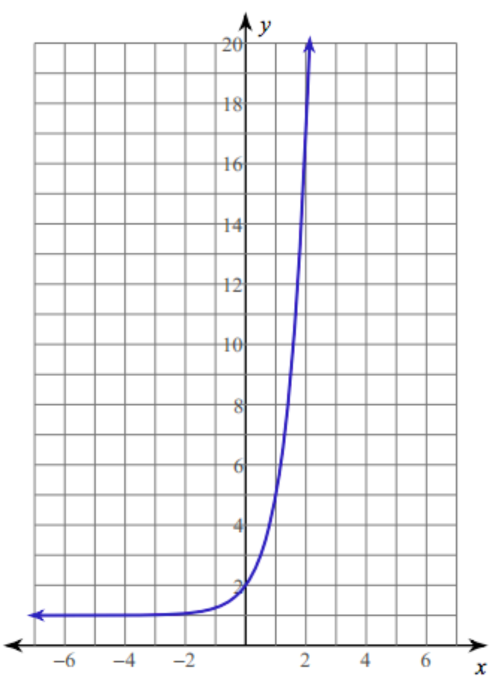


6) $y = 4 \cdot \left(\frac{1}{2}\right)^{x-1} - 2$

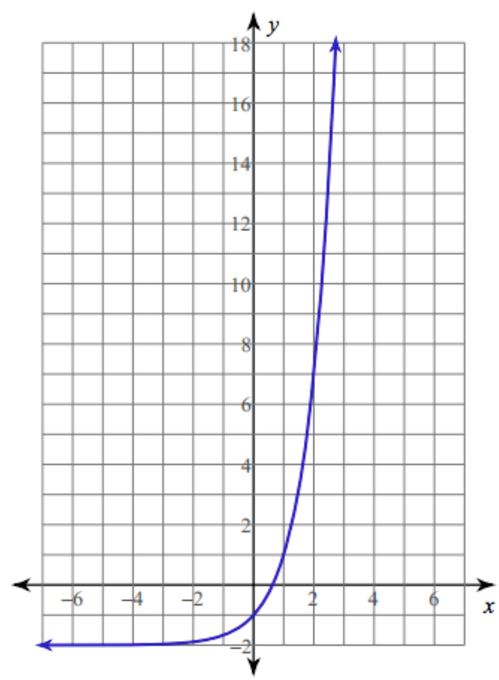


Write an equation for each graph.

7)



8)



PART 2 – Skills PRACTICE

E. Word Problems – Simple vs Compound Interest

Simple Interest: Ex. $1,000 earning 10% p.a simple interest

Simple Interest Data Table:

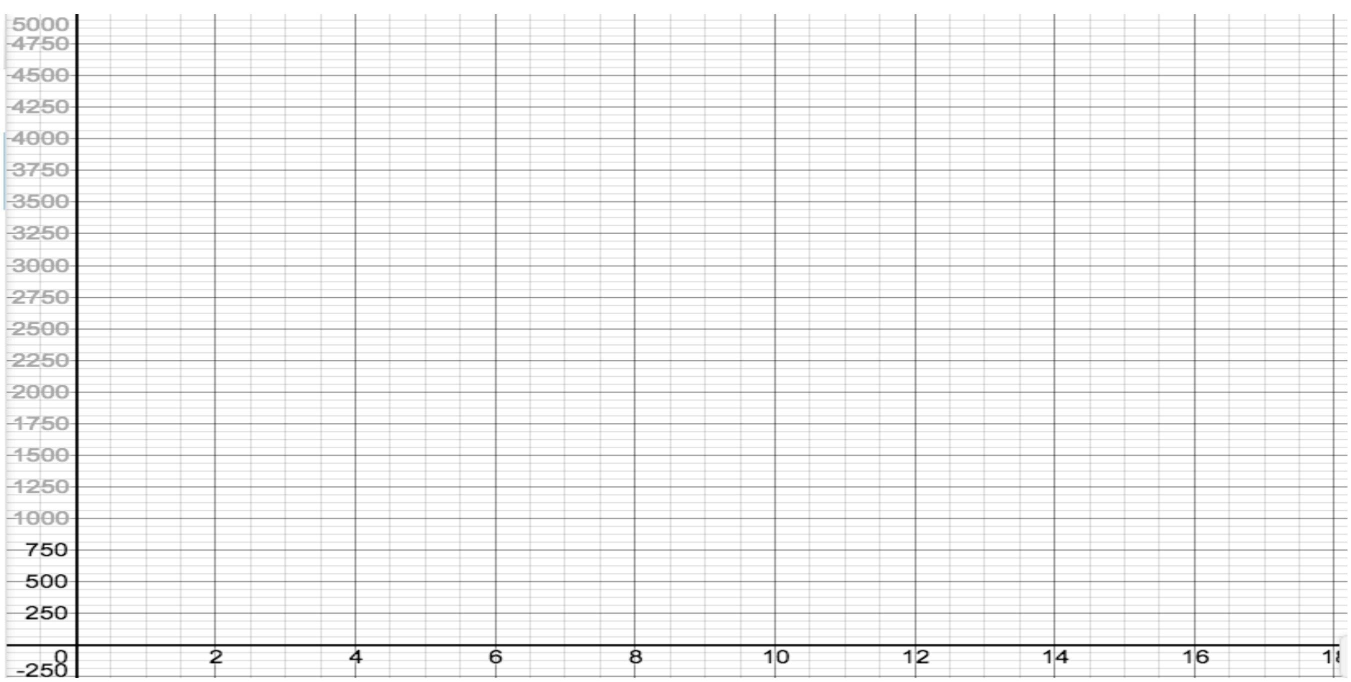
Years	Value:
0	
1	
2	
3	
4	
5	
7	
10	
15	

Compound Interest: Ex. $1,000 earning 10% p.a. compounded annually

Compound Interest Data Table:

Years	Value:
0	
1	
2	
3	
4	
5	
7	
10	
15	

Use your TI-84 to graph each investment option and compare and contrast the two investment options



15. On July 1, 1996, Anna invested $2000 in an account that earned 6%/a compounded monthly. On July 1, 2001, she moved the total amount to a new account that paid 8%/a compounded quarterly. Determine the balance in her account on January 1, 2008.

16. Bernie deposited $4000 into an account that pays 4%/a compounded quarterly during the first year. The interest rate on this account is then increased by 0.2% each year. Calculate the balance in Bernie's account after three years.

17. On the day Rachel was born, her grandparents deposited $500 into a savings account that earns 4.8%/a compounded monthly. They deposited the same amount on her 5th, 10th, and 15th birthdays. Determine the balance in the account on Rachel's 18th birthday.

Extending

19. Liz decides to save money to buy an electric car. She invests $500 every 6 months at 6.8%/a compounded semi-annually. What total amount of money will she have at the end of the 10th year?

9. **A** Franco invests some money at 6.9%/a compounded annually and David invests some money at 6.9%/a compounded monthly. After 30 years, each investment is worth $25 000. Who made the greater original investment and by how much?

10. **T** Sally invests some money at 6%/a compounded annually. After 5 years, she takes the principal and interest and reinvests it all at 7.2%/a compounded quarterly for 6 more years. At the end of this time, her investment is worth $14 784.56. How much did Sally originally invest?

11. Steve wants to have $25 000 in 25 years. He can get only 3.2%/a interest compounded quarterly. His bank will guarantee the rate for either 5 years or 8 years.

- In 5 years, he will probably get 4%/a compounded quarterly for the remainder of the term.
- In 8 years, he will probably get 5%/a compounded quarterly for the remainder of the term.

a) Which guarantee should Steve choose, the 5-year one or the 8-year one?
b) How much does he need to invest?

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For extra practice, complete the three challenge questions from this worksheet:

Extension Questions: QUESTION #1

Mr Smith's wife has just learned that she is pregnant! Mr. Smith wants to know when his new baby will arrive and decides to do some research. On the Internet, he finds the following article:

> **Hormone Levels for Pregnant Women**
>
> When a woman becomes pregnant, the hormone HCG (human chlorionic gonadotropin) is produced in order to enable the baby to develop.
>
> During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. Commonly, the HCG levels are measured two days apart to look for this rate of growth.
>
> A woman who is not pregnant will often have an HCG level of between 0 and 5 mIU (milli-international units) per ml (milliliter).

Then Smith remembered that his wife was tested for HCG during her last two doctor visits.

1. On March 21, her HCG level was 200 mIU/ml, while two days later, her HCG level was 392 mIU/ml.. Assuming that the model for HCG levels is of the form $y = ab^x$, what equation models the growth of HCG for his wife's pregnancy?

2. Assume her HCG level was 5 mIU on the day of implantation. How many days after implantation was his wife's first doctor visit? March 21? What day did the baby most likely become implanted?

3. Smith also learned that a baby is born approximately 37 weeks after implantation. What day can Smith expect to become a father?

QUESTION #2: *<u>Doses of Medicine</u>*

Medicine in the body decays in an exponential way. Mr. Smith is taking some medication. On Monday Mr. Smith took the pills. On Tuesday he had 15 mg of medicine left in his body. On Friday he had 6.328125 mg left in his body.

1. Create an exponential equation modeling this situation. (Remember… starting should be on Monday)

2. When the amount of medicine in Mr. Smith's body drops below 4 mg, he needs to take another pill. When does Mr. Smith need to take more medicine?