A. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

B. Lesson Objectives

i. Study applications of exponential functions

PART 1 – Concept Investigations

C. Working with Rational Exponents

Find the exact, simplified value of each expression **without a calculator**. *If you are stuck, try converting between radical and rational exponential notation first, and then simplify.* Sometimes, simplifying the exponent (or changing a decimal to a fraction) is very helpful.

a. $125^{1/3} =$ b. $64^{-1/2} =$ c. $64^{1/6} =$

d. $81^{1/2} =$ e. $32^{-1/5} =$ f. $81^{-1/4} =$

g. $4^{3/2} =$ h. $(-64)^{2/3} =$ i. $(-8)^{-5/3} =$

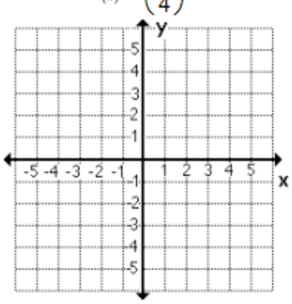
j. $9^{-3/2} =$ k. $\left(\frac{9}{4}\right)^{3/2} =$ l. $16^{-1.5} =$

m. $\left(\sqrt[3]{-27}\right)^2 =$ n. $\sqrt[3]{125^2} =$ o. $\left(\sqrt[3]{4}\right)^6 =$

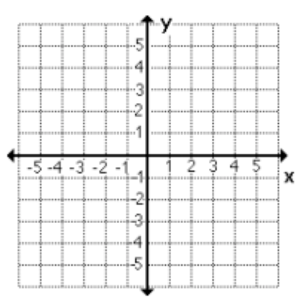
p. $\left(\sqrt{5}\right)^{-2} =$ q. $\left(\sqrt[4]{2}\right)^{-4} =$ r. $\left(\sqrt[5]{3}\right)^5 =$

D. Graphs of Exponential Functions

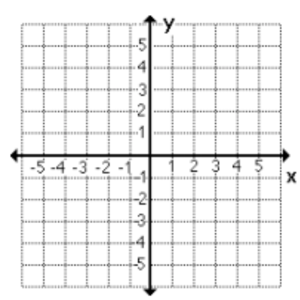
7. $f_{(x)} = \left(\frac{1}{4}\right)^{-x} + 1$



8. $f_{(x)} = 2^{x-2} + 3$



9. $f_{(x)} = -2^{x+3} - 4$



PART 2 – Skills PRACTICE

E. Word Problems

Review ➔ An Exponential equation has the form $\mathbf{y = a(b)^x}$ or $\mathbf{y = a(1 + r)^x}$, where **a** = initial value, **b** is the growth factor/common ratio. (It turns out that **b** = **1 + r** , where **r** is the decimal value of % increase given).

For the following equations, (i) decide if they can be used to model growth or decay and (ii) determine the rate at which the change happens.

(vi) $y = 400(1.75)^x$ (vii) $y = 100(0.75)^x$ (viii) $y = 100(0.995)^x$ (ix) $y = 1{,}000(0.30)^x$ (x) $y = 2500(1.5)^x$

a. A colony of 1,000 ants can increase by 15% in a month.
   i. How many ants will be in the colony after 10 months?
   ii. How long will it take to get 7,500 ants in the colony?

b. A population of 10 hamsters will triple every year.
   i. What will be the population after 4 years?
   ii. How long will it take to get 1,500 hamsters?
   iii. Determine the WEEKLY growth rate for the hamsters.

c. A baby weighing 7 pounds at birth may increase in weight every month according to the function $W(m) = 7(1.11)^m$.
   i. How much will the baby weigh after 1 year?
   ii. When will the baby weigh 18 pounds?
   iii. Determine the monthly rate of growth for this infant.
   iv. Determine the approximate DAILY rate of growth for this infant.

d. A deposit of $1500 in an account pays interest on the balance annually and the account balance is modeled by the function $B(t) = 1500(1.0725)^t$.
   i. Determine the yearly rate of increase of the account balance.
   ii. What is the account balance after 8 years?
   iii. When will the value of the account be double its original value?

e. A colony of 100,000 ants is infected by a virus and its monthly population is modeled by the following function: $P(m) = 100000(0.88)^m$.

    i. How many ants will be in the colony after 10 months?
    ii. How long will it take to get 25,000 ants in the colony?
    iii. Determine the monthly rate of decrease of ant population.
    iv. Determine the DAILY death rate for the ant colony.

f. A sample of 100 g radioactive plutonium-238 has a half-life of 87.7 years, so it will exponentially decay every year.

    i. Determine the YEARLY decay rate for plutonium.
    ii. What amount will remain after 400 years?
    iii. How long will it take to eliminate 95% of the plutonium?

g. An investment of $150,000 in an account loses value at a rate of 3.25% annually.

    i. What is the account balance after 5 years?
    ii. When will the value of the account be half its original value?

F. ***Examining Changes in the Compounding Conditions***

When my oldest son, Alexander, was born, I invested $5,000 in an education fund for him. The education fund is earning 8% compound interest every year. You will develop an answer to my questions

(a) How much this investment is worth when Alexander starts university at the age of 19 years old?

(b) When has the investment tripled its value?

When interest is "paid" to the investor, it DOES NOT HAVE TO BE ANNUALLY!!!. What if an investor (like me) wants the interest paid MORE FREQUENTLY? How does this change the value of an investment?? How does it change the formula that I can use to predict future values?

Let's reconsider my first example: When my oldest son, Alexander, was born, my wife and I invested $5,000 in an education fund for him. The education fund is earning 8% interest every year ➔ Now I will have 4 investment options that you will investigate:

OPTION A ➔ 8%/a compounded semi-annually

OPTION B ➔ 8%/a compounded quarterly

OPTION C ➔ 8%/a compounded monthly

OPTION D ➔ 8%/a compounded daily

(c) Summary

a. Does the value of my investment for Alex change in value given the different compounding conditions? Any ideas as to WHY/WHY NOT?

b. Does the time taken to triple my investment change given the different compounding conditions? Any ideas as to WHY/WHY NOT?

c. Does the formula I use to predict future values change given the different compounding conditions?

(d) Examples ➔ For each situation, determine: (i) the amount (value of the investment) (ii) the interest earned

i. $4000 borrowed for 4 years at 3%/a, compounded annually

ii. $7500 invested for 6 years at 6%/a, compounded monthly

iii. $15 000 borrowed for 5 years at 2.4%/a, compounded quarterly

iv. $28 200 invested for 10 years at 5.5%/a, compounded semi-annually

v. $850 financed for 1 year at 3.65%/a, compounded daily

vi. $2225 invested for 47 weeks at 5.2%/a, compounded weekly