A. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

B. Lesson Objectives

i. Study applications of exponential functions

PART 1 – Concept Investigations

C. Working with Rational Exponents

Find the exact, simplified value of each expression **without a calculator**. *If you are stuck, try converting between radical and rational exponential notation first, and then simplify.* Sometimes, simplifying the exponent (or changing a decimal to a fraction) is very helpful.

a. $8^{2/3} =$

b. $(-27)^{2/3} =$

c. $25^{-3/2} =$

d. $\left(\frac{8}{27}\right)^{-2/3} =$

e. $4^{1.5} =$

f. $\left(\frac{1}{4}\right)^{-1.5} =$

g. $\left(\sqrt[3]{64}\right)^{4} =$

h. $\left(\sqrt{3}\right)^{6} =$

i. $\left(\sqrt[4]{3}\right)^{-8} =$

D. Graphs of Exponential Functions

For Problems 3 – 14, graph each exponential function. State the domain and range for each along with the equation of any asymptotes. Check your graph using a graphing calculator.

3. $f(x) = 3^x$

4. $f(x) = -(3^x)$

5. $f(x) = 3^{-x}$

6. $f(x) = \left(\frac{1}{3}\right)^x$

7. $f(x) = 2^x - 3$

8. $f(x) = 2^{x-3}$

9. $f(x) = 2^{x+5} - 5$

10. $f(x) = -2^{-x}$

PART 2 – Skills PRACTICE

E. Word Problems

Review ➔ An Exponential equation has the form $\mathbf{y = a(b)^x}$ or $\mathbf{y = a(1 + r)^x}$, where **a** = initial value, **b** is the growth factor/common ratio. (It turns out that **b** = **1 + r** , where **r** is the decimal value of % increase given).

For the following equations, (i) decide if they can be used to model growth or decay and (ii) determine the rate at which the change happens.

(i) $y = 200(1.15)^x$ (ii) $y = 400(0.85)^x$ (iii) $y = 100(2)^x$ (iv) $y = 100(\frac{1}{2})^x$ (v) $y = 200(1.05)^x$

Opening Exploration ➔ Mr Santowski has been given a new job contract. He will earn $40,000 per year and get a raise of 6% of his previous years' salary (i.e his salary grows by 6% per year)

a) Define the variables that you will be using to model this problem.
b) Write an equation for Mr. S's salary.
c) Graph the function on your TI-84
d) What does the y-intercept represent?
e) What would my salary be in 8 years?
f) After how many years would my salary be $70,000?
g) What assumption are you making as you answer Qe,f?
h) I would like Mr. S's salary to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

Opening Exploration ➔ Mr Santowski has purchased a new car. It cost $50,000 but its value is depreciating at a rate of 12%.

a) Define the variables that you will be using to model this problem.
b) Write an equation for the value of Mr. S's car.
c) Graph the function on your TI-84.
d) What does the y-intercept represent?
e) What would be the value of my car be in 8 years?
f) After how many years would the value of my car be $7,000?
g) I would like the value of Mr. S's car to be modelled with a linear relation. HOW would you change the original info so that a linear model can be used?

1. A dish has 212 bacteria in it. The population of bacteria will double every 2 days. How many bacteria will be present in . . .
   a. 8 days b. 11 days c. 4 hours d. 2 months

2. An experiment starts off with X bacteria. This population of bacteria will double every 7 days and grows to 11,888 in 32 days. How many bacteria where present at the start of the experiment?

3. A bacteria culture grows according to the formula: $y = 12000(2)^{\frac{t}{4}}$ where t is in hours. How many bacteria are present:
   (a) at the beginning of the experiment?
   (b) after 12 hours?
   (c) after 19 days?
   (d) What is the doubling time of the bacteria?

4. A bacteria culture starts with 3000 bacteria. After 3 hours there are 48 000 bacteria present. What is the length of the doubling period?

5. Mr S. makes an initial investment of $15,000. This initial investment will double every 9 years. What is the value of this investment in . . .
   a. 20 years b. 6 years c. What is the yearly rate of increase of this investment?

6. Iodine-131 is a radioactive isotope of iodine that has a half-life of 8 days. A science lab initially has 200 grams of iodine-131. How much iodine-131 will be present in . . .
   a. 8 days b. 20 days c. 1 year d. 2 months

7. A medical experiment starts off with X grams of a radioactive chemical called Mathematus. This chemical will decay in half every 15 seconds and in the course of the experiment, will decay to 9.765 g in 2 minutes. How much Mathematus was present at the start of the experiment?

8. A chemical decays according to the formula: $y = 12000\left(\frac{1}{2}\right)^{\frac{t}{25}}$ where t is in time in hours and y is amount of chemical left, measured in grams. What amount of chemical is present:

   (e) at the beginning of the experiment?
   (f) after 100 hours?
   (g) after 19 days?
   (h) What is the half-life of the chemical?

9. A block of dry ice is losing its mass at a rate of 12.5% per hour. At 1 PM it weighed 50 pounds. What was its weight at 5 PM? What was the approximate half-life of the block of dry ice under these conditions?