1. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

2. Lesson Objectives

i. Look for patterns in data sets and in context

ii. Create algebraic models to help sumarize and then analyze these data sets

PART 1 – Concept Investigations

(a) Working with Rational Exponents ➔ what does $B^{\frac{1}{n}}$ mean?

EXAMPLE 1 Representing a side length by rearranging the area formula

Express the side length x as a power of A and V.

Ira's Solution

$A = x^2$
$x = A^n$
$A = (x)(x)$

I used the area formula for the base. Since I didn't know what power to use, I used the variable n to write x as a power of A.

$A = A^n \times A^n$
$A = A^{n+n}$
$A^1 = A^{2n}$

I rewrote the area formula, substituting A^n for x.

Since I was multiplying powers with the same base, I added the exponents.

Therefore,

$1 = 2n$

I set the two exponents equal to each other. I solved this equation.

$\frac{1}{2} = n$

The exponent that represents a square root is $\frac{1}{2}$.

Therefore, $x = A^{\frac{1}{2}} = \sqrt{A}$.

EXAMPLE 2 Representing a side length by rearranging the volume formula

Sienna's Solution

$V = x^3$
$x = V^n$
$V = (x)(x)(x)$

I used the volume formula for a cube. I represented the edge length x as a power of the volume V. I used the variable n.

$V = V^n \times V^n \times V^n$

I rewrote the volume formula, substituting V^n for x.

$V = V^{n+n+n}$

I added the exponents.

$V^1 = V^{3n}$

Therefore,

$1 = 3n$

I set the two exponents equal to each other. I solved this equation.

$\frac{1}{3} = n$

The exponent that represents a cube root is $\frac{1}{3}$.

Therefore, $x = V^{\frac{1}{3}} = \sqrt[3]{V}$.

(b) Practice

CHECK Your Understanding

1. Write in radical form. Then evaluate without using a calculator.

a) $49^{\frac{1}{2}}$ **c)** $(-125)^{\frac{1}{3}}$ **e)** $81^{\frac{1}{4}}$

b) $100^{\frac{1}{2}}$ **d)** $16^{0.25}$ **f)** $-(144)^{0.5}$

2. Write in exponent form, then evaluate. Express answers in rational form.

a) $\sqrt[9]{512}$ **c)** $\sqrt[3]{27^2}$ **e)** $\sqrt[5]{\frac{-32}{243}}$

b) $\sqrt[3]{-27}$ **d)** $(\sqrt[3]{-216})^5$ **f)** $\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

3. Write as a single power.

a) $8^{\frac{2}{3}}(8^{\frac{1}{3}})$ **d)** $(7^{\frac{5}{6}})^{-\frac{6}{5}}$

b) $8^{\frac{2}{3}} \div 8^{\frac{1}{3}}$ **e)** $\frac{9^{\frac{-1}{5}}}{9^{\frac{2}{3}}}$

c) $(-11)^2(-11)^{\frac{3}{4}}$ **f)** $10^{-\frac{4}{5}}(10^{\frac{1}{15}}) \div 10^{\frac{2}{3}}$

(c) Working with Functions

i. Use GEOGEBRA to graph the function $f(x) = 2x$.

a. Create a “point on object” ➔ in other words, put a point onto $f(x)$ using the “point on object” tool

b. Use the rotation tools (3rd last tool icon) and use it as follows:

i. Reflect across the x-axis

ii. Reflect across the y-axis

iii. Reflect across the line $y = x$

iv. Rotate 180°

c. Perform these changes on the function $f(x)$ as well as the point. Describe what happens to both the point and the linear function.

d. Write down the new “equation” of the line each time and explain how the equation of the line is related to the change being made.

ii. Use GEOGEBRA to graph the function $g(x) = 2^x$.

a. Create a “point on object” ➔ in other words, put a point onto $g(x)$ using the “point on object” tool

b. Use the rotation tools (3rd last tool icon) and use it as follows:

   i. Reflect across the x-axis
   ii. Reflect across the y-axis
   iii. Reflect across the line $y = x$
   iv. Rotate 180°

c. Perform these changes on the function $g(x)$ as well as the point. Describe what happens to both the point and the exponential function.

d. Write down the new “equation” of the exponential function each time and explain how the equation of the function is related to the change being made.

(d) Working with Data Sets

i. Data Set #1 ➔ Use our data from PS 4.1 on Cancer Cells ➔Heads or Tails Activity ➔ Modeling Exponential Growth H&T Activity

   a. MATH ANALYSIS #1 ➔ Common Ratio
   b. MATH ANALYSIS #2 ➔ Percent Change
   c. Which leads to an equation ➔
   d. Verification ➔ use the TI-84 calculator to verify our equation ➔ HOW???

ii. Data Set #2 ➔ Use our data from PS 4.1 on Radioactive Decay ➔ Radioactivity Simulation➔ Modeling Exponential Decay Activity

   a. MATH ANALYSIS #1 ➔ Use Common Ratio method
   b. MATH ANALYSIS #2 ➔ Use Percent Change method
   c. Equation ➔
   d. Verification ➔ use the TI-84 calculator to verify our equation ➔ HOW???

iii. Data Set #3

a. Describe the pattern in words

b. MATH ANALYSIS #1 ➔ Common Ratio

c. MATH ANALYSIS #2 ➔ Percent Change

d. Equation ➔ ???

e. Verification ➔ use the TI-84 calculator to verify

Year	Population
1950	2.56
1960	3.04
1970	3.71
1980	4.45
1990	5.29
1995	5.780
2000	6.09
2005	6.47
2010	6.90

(e) Generating Data Sets ➔ **Pay It Forward**

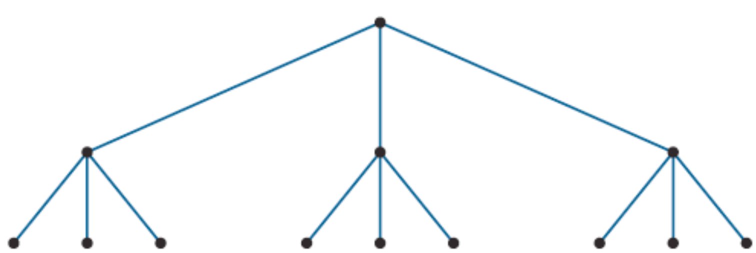
Can you think of an idea for world change, and put it into practice?

Joey came up with an idea that fascinated his mother, his teacher, and his classmates. He suggested that he would do something really good for three people. Then when they ask how they can pay him back for the good deeds, he would tell them to "pay it forward" - each doing something good for three other people.

Joey figured that those three people would do something good for a total of nine others. Those nine would do something good for 27 others, and so on. He was sure that before long there would be good things happening to billions of people all around the world.

Counting in Tree Graphs

The number of good deeds in the Pay It Forward pattern can be represented by a tree graph that starts like this:



The vertices represent the people who receive and do good deeds. Each edge represents a good deed done by one person for another.

Problem 1.

At the start of the Pay It Forward process, only one person does good deeds - for three new people. In the next stage, the three new people each do good things for three more new people. In the next stage, nine people each do good things for three more new people, and so on, with no person receiving more than one good deed.

a. Make a table that shows the number of people who will receive good deeds at each of the next seven stages of the Pay It Forward process. Then plot the (stage, number of good deeds) data.

Stage	1	2	3	4	5	6	7	8	9	10
Number of Good Deeds	3	9	27							

b. How does the number of good deeds at each stage grow as the tree progresses? How is that pattern of change shown in the plot of the data?

c. How many stages of the Pay It Forward process will be needed before a total of at least 25,000 good deeds will be done?

Problem 2.

Consider now how the number of good deeds would grow if each person touched by the Pay It Forward process were to do good deeds for only two other new people, instead of three.

a. Make a tree graph for several stages of this Pay It Forward process.

b. Make a table showing the number of good deeds done at each of the first 10 stages of the process and plot those sample (stage, number of good deeds) values.

c. How does the number of good deeds increase as the Pay It Forward process progresses in stages? How is that pattern of change shown in the plot of the data?

d. How many stages of this process will be needed before a total of 25,000 good deeds will have been done?

Problem 3.

In the two versions of Pay It Forward that you have studied, you can use the number of good deeds at one stage to calculate the number at the next stage.

a. Use the words NOW and NEXT to write rules that express the two patterns. That is, rules of the form NEXT = …

b. How do the numbers and calculations indicated in the rules express the patterns of change in tables of (stage, number of good deeds) data?

c. Write a rule relating NOW and NEXT that could be used to model a Pay It Forward process in which each person does good deeds for four other new people. What pattern of change would you expect to see in a table of (stage, number of good deeds) data for this Pay It Forward process?

Problem 4.

It is also convenient to have rules that will give the number of good deeds N at any stage x of the Pay It Forward process, without finding all the numbers along the way to stage x.

When students in one class were given the task of finding such a rule for the process in which each person does three good deeds for others, they came up with four different ideas:

$N = 3x$
$N = x + 3$
$N = 3^x$
$N = 3x + 1$

a. Are any of these rules for predicting the number of good deeds N correct? How do you know?

b. How can you be sure that the numbers and calculations expressed in the correct "N = ..." rule will produce the same results as the NOW-NEXT rule you developed in Problem 3?

c. Write an "N = ..." rule that would show the number of good deeds at stage number x if each person in the process does good deeds for two others.

d. Write an "N = ..." rule that gives the number of good deeds at stage x if each person in the process does good deeds for four others

PART 2 – Skills PRACTICE

From the [Nelson 11 textbook, Chap 4.3](#) ➔ starting on page 229, Q5,6,8,10,11,13