1. Lesson Context

BIG PICTURE of this UNIT:	• How can we analyze growth or decay patterns in data sets & contextual problems? • How can we algebraically & graphically summarize growth or decay patterns? • How can we compare & contrast linear and exponential models for growth and decay problems. • How can we extend basic function concepts using exponential functions?

2. Lesson Objectives

i. Look for patterns in data sets and in context
ii. Create algebraic models to help sumarize and then analyze these data sets

PART 1 – Concept Investigations

(a) Working with Exponents.

2. Use the exponent laws to write each expression with a single, simplified base.

a) $x^4 \cdot x^5 \cdot x^9$ c) $\frac{x^{12}}{x^4}$ e) $\frac{a}{a^{-5}}$ g) $\frac{(k^a)^b \cdot k^{3ab}}{k^{7ab}}$

b) $x^4 \cdot x^{-5}$ d) $\frac{a^{10}}{a^{14}}$ f) $(g^7)^{20}$ h) $(\sqrt{x})^6$

3. Evaluate (simplify as a number) the following.

a) -3^2 f) $\left(\frac{-2}{5}\right)^2$

b) $(-3)^2$ g) $\left(\frac{-2}{5}\right)^{-2}$

c) -3^{-2} h) $\left[\left(\frac{-2}{5}\right)^{-2}\right]^{-1}$

d) $(-3)^{-2}$ i) $-\left(\frac{-2}{5}\right)^2$

e) $(3^{-2}+3^{-3})^{-1}$ j) $\left(\frac{-2}{5}\right)^3$

(b) Working with Functions

i. Use GEOGEBRA to graph the function $f(x) = 2x$.

a. Create a "point on object" ➔ in other words, put a point onto $f(x)$ using the "point on object" tool
b. Create a vector
c. Now use the "translate by vector" tool and apply it to the function $f(x)$ as well as the point. Describe what happens to both the point and the linear function.
d. Write down the new "equation" of the line and explain how the equation of the line is related to the translation vector.

ii. Use GEOGEBRA to graph the function $g(x) = 2^x$.

a. Create a "point on object" ➔ in other words, put a point onto $g(x)$ using the "point on object" tool
b. Create a vector
c. Now use the "translate by vector" tool and apply it to the function $g(x)$ as well as the point. Describe what happens to both the point and the exponential function.
d. Write down the new "equation" of the line and explain how the equation of the exponential function is related to the translation vector.

(c) Working with Data Sets

i. Data Set #1

x	0	1	2	3	4	5	6
y	1	2	4	8	16	32	64

a. Describe the pattern in words

b. MATH ANALYSIS #1 ➔ Common Ratio ➔ To calculate the common ratio, we will divide successive y values.

$$ratio = \frac{y_2}{y_1} = \frac{y_3}{y_2} = \frac{y_4}{y_3} = \frac{y_5}{y_4} \quad \text{etc ......} \quad \Rightarrow \text{observation ..... ?}$$

Which leads to an equation ➔ $f(x) = ab^x$ ➔

c. MATH ANALYSIS #2 ➔ Percent Change ➔ To calculate the percentage, we will calculate the percent change for each trial using the formula below.

$$percentage\ change = r = \frac{y_2 - y_1}{y_1} = \frac{y_3 - y_2}{y_2} = \frac{y_4 - y_3}{y_3} = \frac{y_5 - y_4}{y_4} = \text{etc ....} \quad \Rightarrow \text{observation ..... ?}$$

Which leads to an equation ➔ $f(x) = a(1 + r)^x$ ➔

d. Verification ➔ use the TI-84 calculator to verify our equation ➔ HOW???

ii. Data Set #2

x	0	1	2	3	4	5	6
y	320	160	80	40	20	10	5

a. Describe the pattern in words

b. MATH ANALYSIS #1 ➔ Use Common Ratio method

c. MATH ANALYSIS #2 ➔ Use Percent Change method

d. Equation ➔

e. Verification ➔ use the TI-84 calculator to verify our equation ➔ HOW???

iii. Data Set #3

Year	Population
1700	250
1750	370
1800	560
1850	840
1900	1270
1950	1900
2000	2850

a. Describe the pattern in words

b. MATH ANALYSIS #1 ➔ Common Ratio

c. MATH ANALYSIS #2 ➔ Percent Change

d. Equation ➔ ???

e. Verification ➔ use the TI-84 calculator to verify

iv. Data Set #4

Year	Value
2002	40
2003	36
2004	32.4
2005	29.2
2006	26.2
2007	23.6
2008	21.3
2009	19.1
2010	17.2

a. Describe the pattern in words

b. MATH ANALYSIS #1 ➔ Common Ratio

c. MATH ANALYSIS #2 ➔ Percent Change

d. Equation ➔ ???

e. Verification ➔ use the TI-84 calculator to verify

(d) Generating Data Sets ➔ **PAPER FOLDING: Getting to the Moon**

In this simulation activity, you will predict how many times you fold a piece of paper in order to get a tall enough piece of folded paper that reaches to the moon.

PREDICTION: how many times can you fold a piece of A4 paper, so that the resulting height of the folded piece of paper reaches to the moon? ____________________.

ACTIVITY: Follow these steps and answer the questions asked.

a. In trial #0, you simply have 1 sheet of paper (data point of (0,1) is already recorded for you.

b. For trial #1, you will fold your paper in half (so in other words, you now have folded the original sheet for the first time). In our simulation, place 2 full sheets in a stack on your table, one on top of the other.

c. For trial #2, you will fold your paper in half again (so in other words, you now have folded the original sheet for the second time). In our simulation, place another 2 full sheets on your stack, one on top of the other. How many sheets do you now have? ________________.

d. For trial #3, you will fold your paper in half again (so in other words, you now have folded the original sheet for the third time). In our simulation, place another 4 full sheets on your table, one on top of the other. How many sheets do you now have? ________________.

e. For trial #4, you will fold your paper in half again (so in other words, you now have folded the original sheet for the fourth time). In our simulation, place another 8 full sheets on your table, one on top of the other. How many sheets do you now have? ________________.

f. For trial #5, you will fold your paper in half again (so in other words, you now have folded the original sheet for the fifth time). In our simulation, place another 16 full sheets on your stack, one on top of the other. How many sheets do you now have? ________________.

g. For trial #6, you will fold your paper in half again (so in other words, you now have folded the original sheet for the sixth time). In our simulation, place another 32 full sheets on your table, one on top of the other. How many sheets do you now have? ________________.

h. For trial #7, you will fold your paper in half again (so in other words, you now have folded the original sheet for the seventh time). In our simulation, place 64 full sheets on your table, one on top of the other. How many sheets do you now have? ________________.

i. For trial #8, you will fold your paper in half again (so in other words, you now have folded the original sheet for the eighth time). In our simulation, place another 128 full sheets on your table, one on top of the other. How many sheets do you now have? ________________.

j. You now need some data/information from the internet. What data/information do you need?

k. Record the information: ______________________________&______________________________.

# of folds	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Sheets of Paper	1														
Height															

l. You now need to make some calculations ➔

m. So make your final prediction ➔ how many times do you need to fold a sheet of A4 paper in order to get a height equal to the earth-moon distance?

(e) Generating Data Sets #2 ➔ **<u>Grains of Rice Challenge ➔ Legend of the Ambalappuzha Paal Payasam</u>**

There is a well-known story of the man who invented chess. The local ruler was so pleased with the invention that he offered the inventor a great reward in gold. The inventor suggested an alternative reward: he would get one grain of rice on the first square of the chess board, two grains on the second square, four on the third, eight on the fourth, etc., doubling the number of grains each time. The ruler saw that this must be a much better deal for him, and accepted. The board has 64 squares.

a. How many total grains of rice did the ruler have to pay the inventor? Show your work.

b. If these grains of rice were lines up end to end, how far would the line go? Show your work and internet data/information you needed to come up with an estimate.

c. If these grains of rice were used to cover up the land in India, how deep would the pile be? Show your work and internet data/information you needed to come up with an estimate.

The Legend of the Ambalappuzha Paal Payasam is an alternate version of the same story. Check it out!

PART 2 – Skills PRACTICE

From the Nelson 9 textbook, Chap 2.2 ➔ starting on page 90, Q10,11,12,13,14

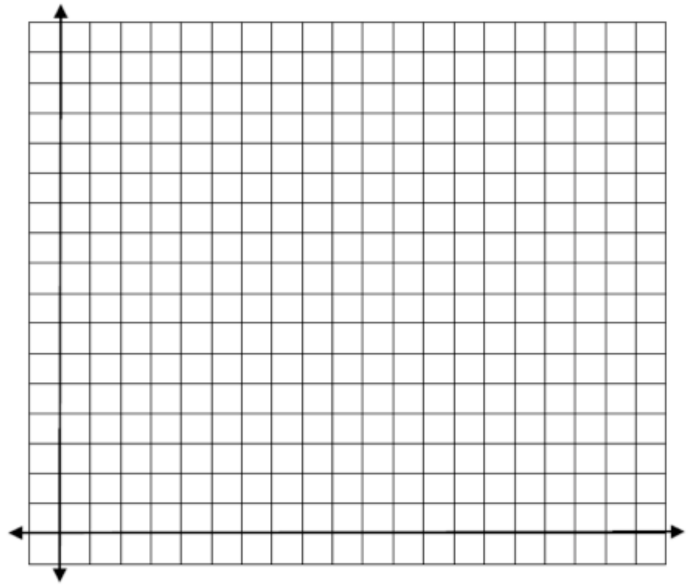
From the Nelson 9 textbook, Chap 2.3 ➔ starting on page 96, Q9,10,11,13,14

DATA SET ANALYSIS #2

Data Set #2 ➔ {5,10,20,40,80,160,320,….} ➔as a data table ➔

X	0	1	2	3	4	5	6
y	5	10	20	40	80	160	320

Describe the pattern in words



MATH ANALYSIS ➔ Common Ratio

Option #1: ➔ To calculate the common ratio, we will divide successive y values.

$$ratio = \frac{y_2}{y_1} = \frac{y_3}{y_2} = \frac{y_4}{y_3} = \frac{y_5}{y_4} \quad \text{etc ......}$$

➔ observation ….. ?

Which leads to an equation ➔ $y = ab^x$

MATH ANALYSIS ➔ Percent Change

Option #2: ➔ To calculate the percentage, we will calculate the percent change for each trial using the formula below.

$$percentage\ change = r = \frac{y_2 - y_1}{y_1} = \frac{y_3 - y_2}{y_2} = \frac{y_4 - y_3}{y_3} = \frac{y_5 - y_4}{y_4} = \text{ etc ....}$$

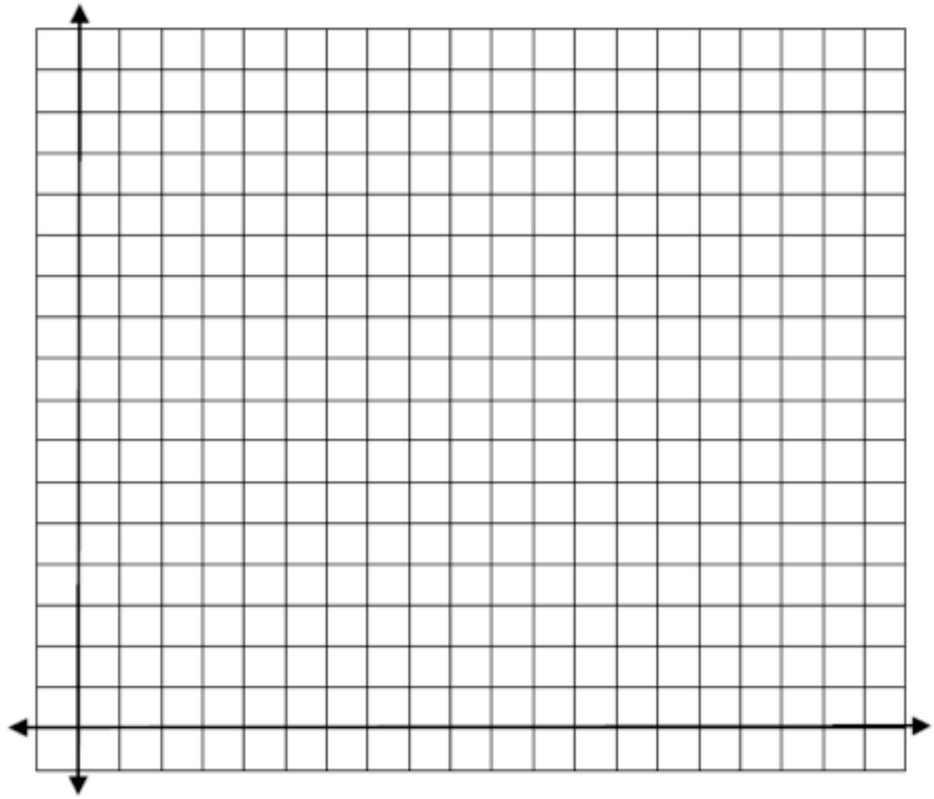
➔ observation ….. ?

Which leads to an equation ➔ $y = a(1+r)^x$ ➔

VERIFICATION ➔ use the TI-84 calculator to verify our equation:

DATA SET ANALYSIS #3

Year	Population
1700	250
1750	370
1800	560
1850	840
1900	1270
1950	1900
2000	2850



MATH ANALYSIS ➔ Common Ratio

Option #1: ➔ To calculate the common ratio, we will divide successive y values.

Calculate the average of ALL the ratios:

____

Which leads to an equation ➔ $y = ab^x$

MATH ANALYSIS ➔ Percent Change

Option #2: ➔ To calculate the percentage, we will calculate the percent change for each trial using the formula below.

Calculate the average of ALL the percents:

____

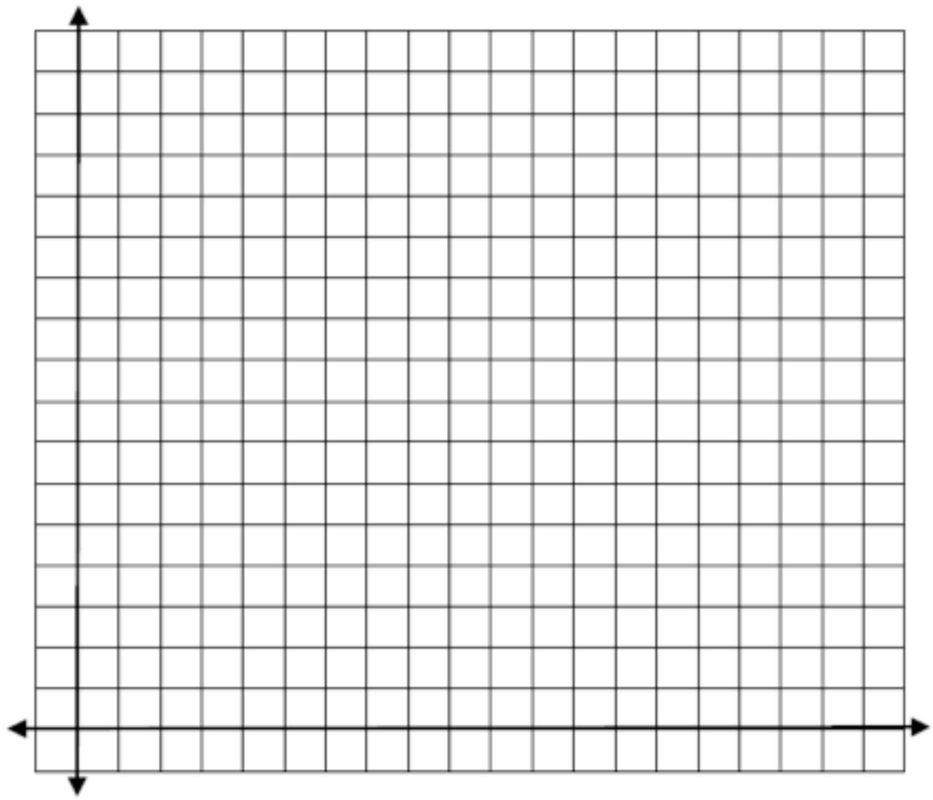
Which leads to an equation ➔ $y = a(1+r)^x$ ➔

VERIFICATION ➔ use the TI-84 calculator to verify our equation:

(A) <u>Data Analysis</u> ➔ <u>*Part I: Modeling Exponential Data*</u>

The value of Mr S car is depreciating over time. I bought the car new in 2002 and the value of my car (in thousands) over the years has been tabulated below:

Year	**Value**
2002	40
2003	36
2004	32.4
2005	29.2
2006	26.2
2007	23.6
2008	21.3
2009	19.1
2010	17.2



MATH ANALYSIS ➔ Common Ratio
Option #1: ➔ To calculate the common ratio, we will divide successive y values.
Calculate the average of ALL the ratios:

____

Which leads to an equation ➔ $y = ab^x$

MATH ANALYSIS ➔ Percent Change
Option #2: ➔ To calculate the percentage, we will calculate the percent change for each trial using the formula below.
Calculate the average of ALL the percents:

____

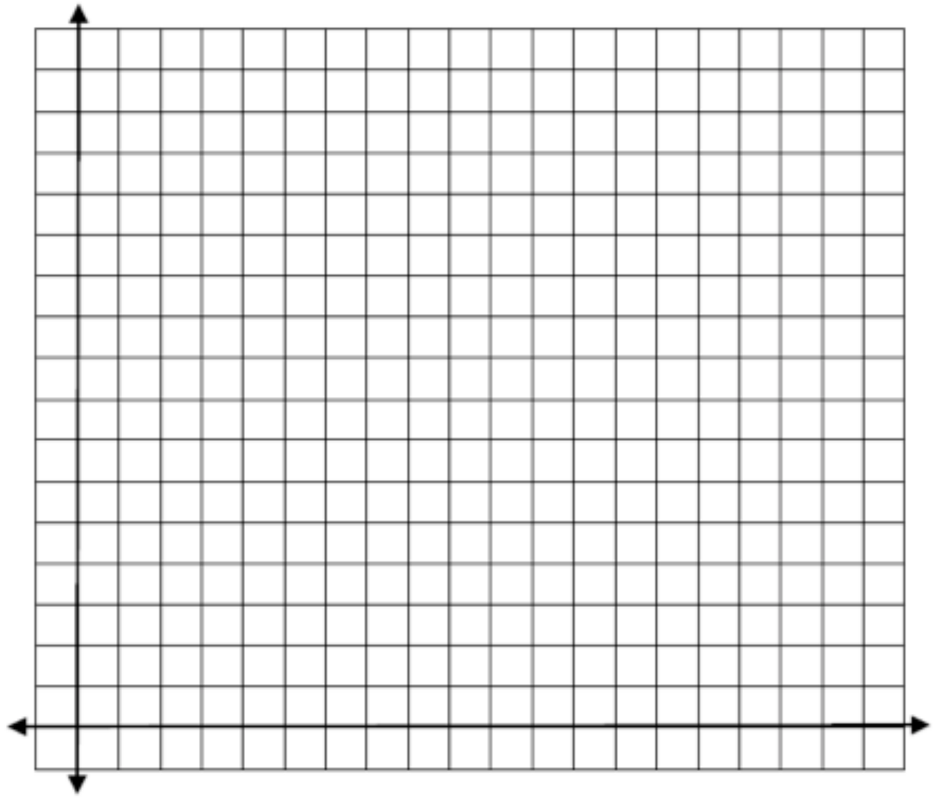
Which leads to an equation ➔ $y = a(1+r)^x$ ➔

VERIFICATION ➔ use the TI-84 calculator to verify our equation:

(B) <u>DATA ANALYSIS ➔ *Part II: Modeling Exponential Data*</u>

The following data table shows the historic world population since 1950:

Year	**Population**
1950	2.56
1960	3.04
1970	3.71
1980	4.45
1990	5.29
1995	5.780
2000	6.09
2005	6.47
2010	6.90



MATH ANALYSIS ➔ Common Ratio

Option #1: ➔ To calculate the common ratio, we will divide successive y values.

Calculate the average of ALL the ratios:

____

Which leads to an equation ➔ $y = ab^x$

MATH ANALYSIS ➔ Percent Change

Option #2: ➔ To calculate the percentage, we will calculate the percent change for each trial using the formula below.

Calculate the average of ALL the percents:

____

Which leads to an equation ➔ $y = a(1+r)^x$ ➔

VERIFICATION ➔ use the TI-84 calculator to verify our equation:

i. Data Set 1 ➔

x	0	1	2	3	4	5
y	5	10	20	40	80	160

a. Describe the pattern in words
b. List the next 6 numbers that you predict would be in the same data set.
c. Find the 25th number in the data set

ii. Data Set 2

x	0	1	2	3	4	5
y	10	13	16	19	22	25

a. Describe the pattern in words
b. List the next 6 numbers that you predict would be in the same data set.
c. Find the 25th number in the data set

iii. Data Set 3

x	0	1	2	3	4	5
y	10000	5000	2500	1250	625	312.5

a. Describe the pattern in words
b. List the next 6 numbers that you predict would be in the same data set.
c. Find the 25th number in the data set

iv. Data Set 4

x	0	1	2	3	4	5
y	120	110	100	90	80	70

a. Describe the pattern in words
b. List the next 6 numbers that you predict would be in the same data set.
c. Find the 25th number in the data set

(f) Generating Data Sets

i. Heads or Tails Activity ➔ Modeling Exponential Growth H&T Activity. The purpose of this activity is to provide a simple model to illustrate exponential growth of cancerous cells. In our experiment, a HEAD on a COIN TOSS represents a cancerous cell. If the COIN lands HEADS side up, the cell divides into the "parent" cell and "daughter" cell. The cancerous cells divide like this uncontrollably-without end. We will conduct 10 trials and record the number of "cancerous cells".

Exponential Growth Procedure

a. Use either the website http://www.shodor.org/interactivate/activities/Coin/ OR https://www.random.org/coins/ to toss our coins. We will start with 2 coins. This is trial # 0.

b. Count the number of HEADS that appear (recall these are cancerous cells) For every coin with the HEAD side showing, add another coin and then record the new population. (Ex. If 5 coins land HEADS, then you add 5 more coins)

c. Repeat step number 2 until you are done with 10 trials.

d. Add your results to this class data table. Use the GROWTH ACTIVITY spreadsheet. https://docs.google.com/spreadsheets/d/18l6kKZd0CcbYl24-o7Quxcwf_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing

Table of Results

Trial #	0	1	2	3	4	5	6	7	8	9	10
# of coins	2										

Prediction #1 ➔ What would you predict for the # of coins for trials 11 and 12? Make your prediction and then test it out.

Prediction #2 ➔ If our “cancer” becomes detectable when there are 10,000 cells, how many trials of our experiment would this take?

(g) Generating Data Sets

i. Radioactivity Simulation➔ Modeling Exponential Decay Activity. The purpose of this activity is to provide a simple model to illustrate exponential decay of radioactive material. In our experiment, let's say that Mrs Knox accidentally spilt some radioactive molecules in her lab, so our building is now UNSAFE and we must evacuate. So, to simulate the decay of a radioactive material, a DICE ROLL of 6 represents a DECAY activity i.e a molecule "changes" form ➔from an "unsafe radioactive form" to a "safe non-radioactive form". If the DICE lands showing a 6, the molecule decays into a non-radioactive form. We will conduct up to 15 trials and record the number of remaining "unsafe radioactive molecules".

Exponential Decay Procedure

a. Use the website http://www.roll-dice-online.com/ to roll our dice. We will start with 99 dice. This is trial # 0.

b. Count the number of 6s that appear (recall these are safe non-radioactive molecules) For every dice showing a 6, remove another dice and then record the new population. (Ex. If 5 dice showing 6s, then you remove 5 dice)

e. Repeat step number 2 until you are done with 10 trials.

f. Add your results to a class data table. Use the GROWTH DECAY spreadsheet. https://docs.google.com/spreadsheets/d/18l6kKZd0CcbY124-o7Quxcwf_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing

Table of Results

Trial #	0	1	2	3	4	5	6	7	8	9	10
# of dice	99										

Prediction #1 ➔ What would you predict for the # of dice for trials 11 and 12? Make your prediction and then test it out.

Prediction #2 ➔ Let's say that our scenario becomes safe OVERALL when there are only 2 unsafe, radioactive molecules left, how many trials of our experiment would this take?

(f) CAC Payment Options

Mr. Rutherford is offering Mr. Santowski & Mr. Smith new contract options for the New Year. Here are the terms of the contracts being offered:

OPTION A ➔ Here is Mr. Smith's payment option: Get paid $5,000 US per day for each day in the month of January.

OPTION B ➔ Here is Mr. Santowski payment option:

1. Get paid 1 piastre on the first day of January.
2. But then on the 2nd of January, return the 1 piastre and get paid double yesterday's wage, so get 2 piastres for having worked 2 days.
3. Now, on the 3rd of January, return the 2 piastres and get paid double yesterday's wage of 2 piastres, making it a total of 4 piastres pay for these three days.
4. Alas, on the 4th of January, return the 4 piastres and get paid double yesterday's wage of these 4 piastres, making it a total of 8 piastres pay for these four days.
5. Oh, woe is me. On the 5th of January, I return the 8 piastres, but get paid double yesterday's wage of these 8 piastres, making it a total of 16 piastres pay for these five days.

   a. Which option would you choose and why?
   b. Are the salaries ever equal? If so when? If not why not?
   c. How much does each Math teacher get paid by the end of January? Convert to a common currency & show your work.

PART 2 – Skills PRACTICE

From the Nelson 9 textbook, Chap 2.2 ➔ starting on page 90, Q6,7,8,9,12

From the Nelson 9 textbook, Chap 2.3 ➔ starting on page 96, Q3,4,6,8