

**1. Lesson Context**

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>• How can we analyze growth or decay patterns in data sets &amp; contextual problems?</li> <li>• How can we algebraically &amp; graphically summarize growth or decay patterns?</li> <li>• How can we compare &amp; contrast linear and exponential models for growth and decay problems.</li> <li>• How can we extend basic function concepts using exponential functions?</li> </ul>
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**2. Lesson Objectives**

- i. Look for patterns in data sets and in context
- ii. Create algebraic models to help summarize and then analyze these data sets

**PART 1 – Concept Investigations**

(a) Working with Exponents.

- i. (CI) Simplify the following algebraic expressions. Your answer should contain only positive exponents

$(x^{-2}x^{-4})^3$	$(x^3)^{-4} \cdot 2x^3$	$(a^2)^2 \cdot 2a^{-2}$	$(2d)^2 \cdot 2d^2$
$(3d)^{-2} \cdot 3d^2$	$\frac{2x^2y^4 \cdot 4x^2y^4}{3x^{-2}y^3}$	$\frac{x^3y^3 \cdot x^2}{4x^3}$	$\frac{2x^2y^4}{3x^{-2}y^3 \cdot 4x^4y^4}$

- ii. (CI) Evaluate the following numerical expressions. All final answers should be written in fraction form (if applicable)

$4^{-3}$	$-5^{-2}$	$5^0$	$10^{-2}$
$-4^{-3}$	$(-4)^{-3}$	$2^{-4}$	$\frac{1}{2^{-2}}$

(b) Working with Functions

- i. Use your TI-84 to graph the function  $f(x) = 2x$ . Prepare a table of values from  $x = -4$  up to  $x = 4$  and then sketch the function.
- ii. Use your TI-84 to graph the function  $g(x) = 2^x$ . Prepare a table of values from  $x = -4$  up to  $x = 4$  and then sketch the function on the same axes as  $f(x)$ .
- iii. Compare the table of values and comment on the patterns that you observe in the function output values

## (c) Working with Data Sets

## i. Data Set 1 →

$x$	0	1	2	3	4	5
$y$	5	10	20	40	80	160

- Describe the pattern in words
- List the next 6 numbers that you predict would be in the same data set.
- Find the 25<sup>th</sup> number in the data set

## ii. Data Set 2

$x$	0	1	2	3	4	5
$y$	10	13	16	19	22	25

- Describe the pattern in words
- List the next 6 numbers that you predict would be in the same data set.
- Find the 25<sup>th</sup> number in the data set

## iii. Data Set 3

$x$	0	1	2	3	4	5
$y$	10000	5000	2500	1250	625	312.5

- Describe the pattern in words
- List the next 6 numbers that you predict would be in the same data set.
- Find the 25<sup>th</sup> number in the data set

## iv. Data Set 4

$x$	0	1	2	3	4	5
$y$	120	110	100	90	80	70

- Describe the pattern in words
- List the next 6 numbers that you predict would be in the same data set.
- Find the 25<sup>th</sup> number in the data set

(d) Generating Data Sets

- i. Heads or Tails Activity → Modeling Exponential Growth H&T Activity. The purpose of this activity is to provide a simple model to illustrate exponential growth of cancerous cells. In our experiment, a HEAD on a COIN TOSS represents a cancerous cell. If the COIN lands HEADS side up, the cell divides into the “parent” cell and “daughter” cell. The cancerous cells divide like this uncontrollably-without end. We will conduct 10 trials and record the number of “cancerous cells”.

Exponential Growth Procedure

- a. Use either the website <http://www.shodor.org/interactivate/activities/Coin/> OR <https://www.random.org/coins/> to toss our coins. We will start with 2 coins. This is trial # 0.
- b. Count the number of HEADS that appear (recall these are cancerous cells) For every coin with the HEAD side showing, add another coin and then record the new population. (Ex. If 5 coins land HEADS, then you add 5 more coins)
- c. Repeat step number 2 until you are done with 10 trials.
- d. Add your results to [this class data table](#). Use the GROWTH ACTIVITY spreadsheet.

[https://docs.google.com/spreadsheets/d/18l6kKZd0CcbY124-o7Quxcwf\\_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing](https://docs.google.com/spreadsheets/d/18l6kKZd0CcbY124-o7Quxcwf_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing)

Table of Results

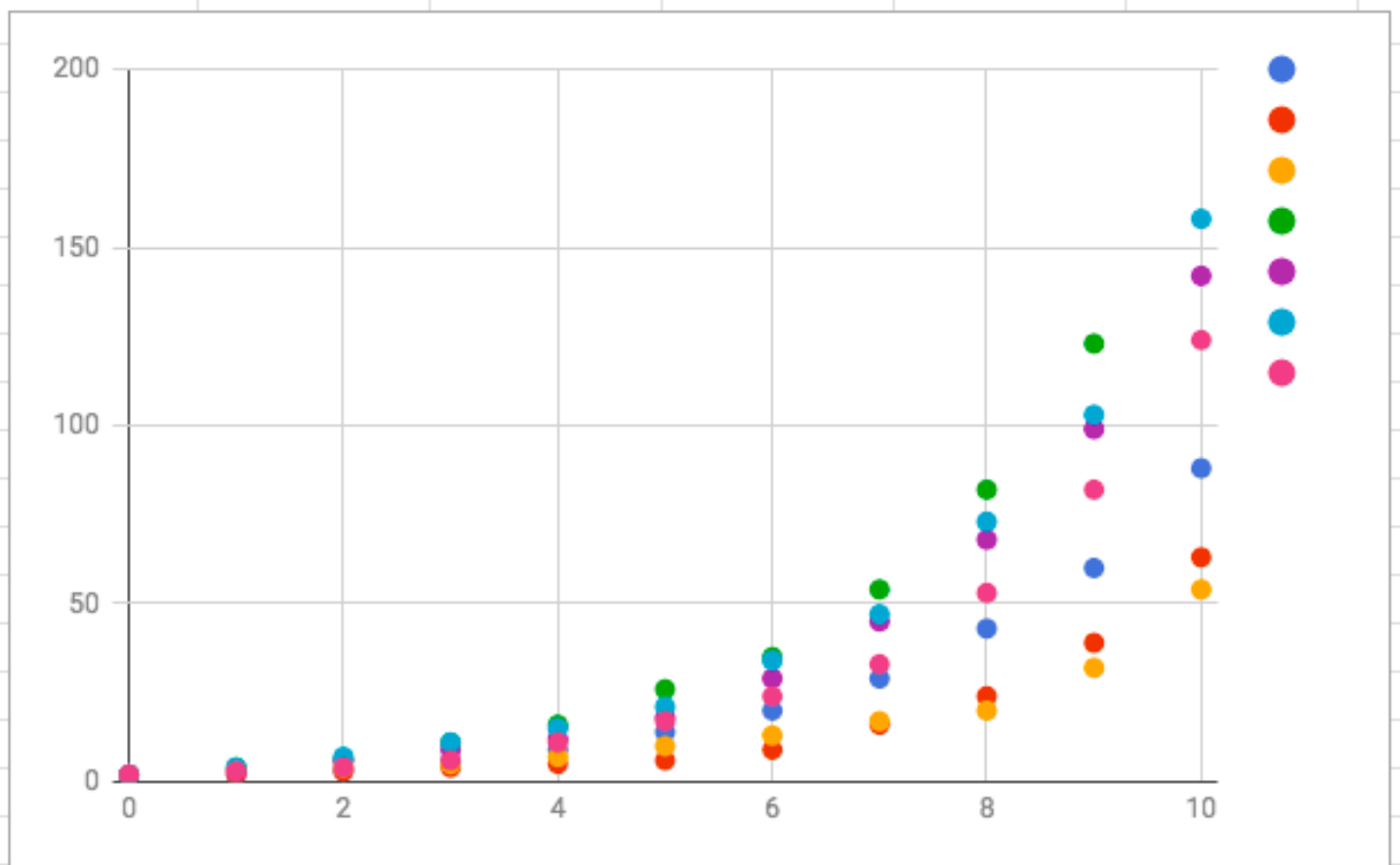
Trial #	0	1	2	3	4	5	6	7	8	9	10
# of coins	2										

Prediction #1 → What would you predict for the # of coins for trials 11 and 12? Make your prediction and then test it out.

Prediction #2 → If our “cancer” becomes detectable when there are 10,000 cells, how many trials of our experiment would this take?

Experimental Results from IM2 2017/2018 class:

Trial #	Group Name/#	Abdhi and Nik Jehe	Youssef Hassan Jana	Taymour and omackson and Jaeholice, Esther, Saks	Liam & Ahmed
0	2	2	2	2	2
1	2	2	3	4	3
2	3	3	4	6	4
3	6	4	5	11	6
4	9	5	7	16	11
5	14	6	10	26	17
6	20	9	13	35	24
7	29	16	17	54	33
8	43	24	20	82	53
9	60	39	32	123	82
10	88	63	54	142	124



(e) Generating Data Sets

- i. Radioactivity Simulation → Modeling Exponential Decay Activity. The purpose of this activity is to provide a simple model to illustrate exponential decay of radioactive material. In our experiment, let’s say that Mrs Knox accidentally spilt some radioactive molecules in her lab, so our building is now UNSAFE and we must evacuate. So, to simulate the decay of a radioactive material, a DICE ROLL of 6 represents a DECAY activity i.e a molecule “changes” form → from an “unsafe radioactive form” to a “safe non-radioactive form”. If the DICE lands showing a 6, the molecule decays into a non-radioactive form. We will conduct up to 15 trials and record the number of remaining “unsafe radioactive molecules”.

Exponential Decay Procedure

- a. Use the website <http://www.roll-dice-online.com/> to roll our dice. We will start with 99 dice. This is trial # 0.
- b. Count the number of 6s that appear (recall these are safe non-radioactive molecules) For every dice showing a 6, remove another dice and then record the new population. (Ex. If 5 dice showing 6s, then you remove 5 dice)
- e. Repeat step number 2 until you are done with 10 trials.
- f. Add your results to a [class data table](#). Use the GROWTH DECAY spreadsheet.

[https://docs.google.com/spreadsheets/d/18l6kKZd0CcbY124-o7Quxcwf\\_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing](https://docs.google.com/spreadsheets/d/18l6kKZd0CcbY124-o7Quxcwf_MRVSXZdWj3qb1Vn1GU/edit?usp=sharing)

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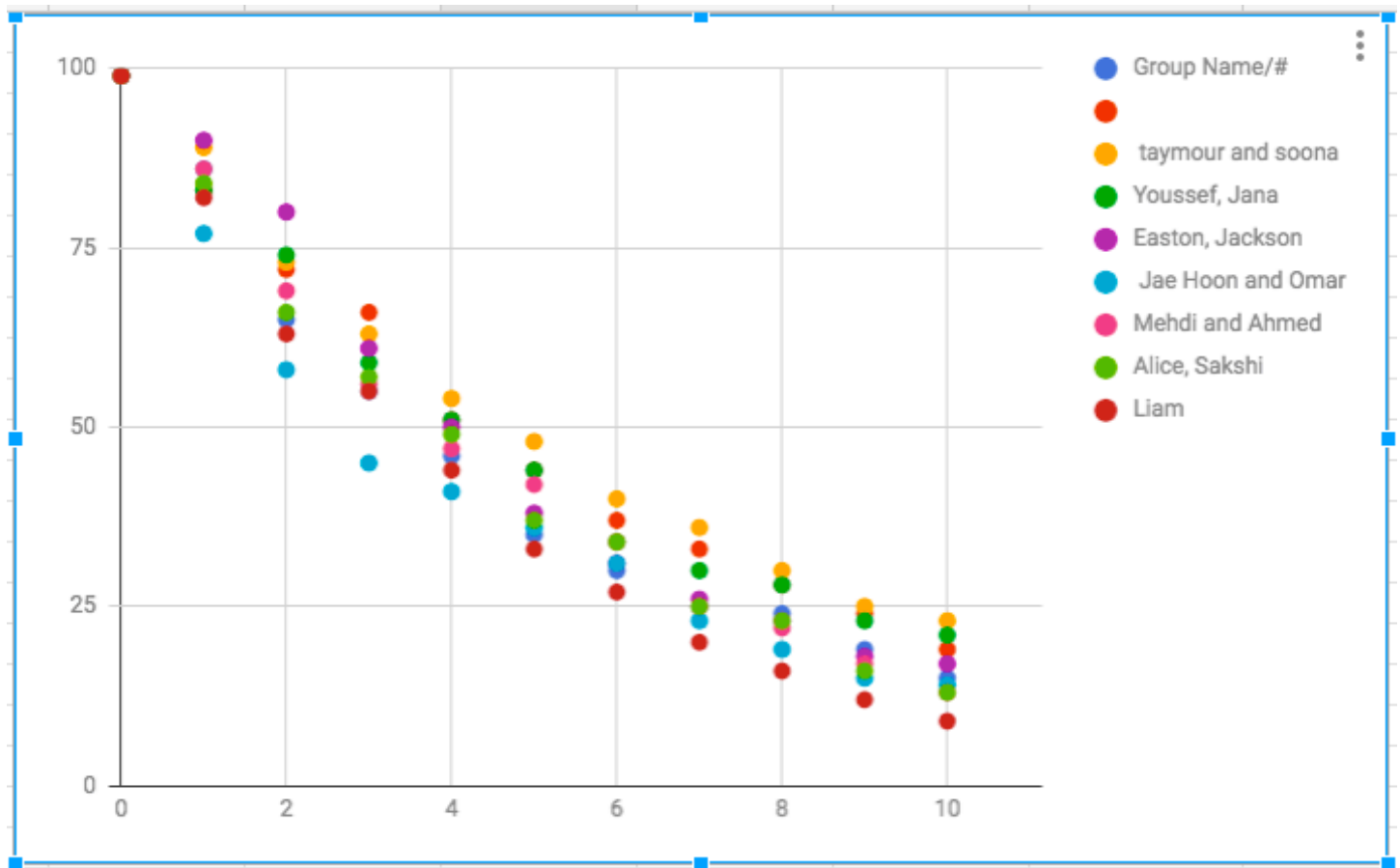
Trial #	0	1	2	3	4	5	6	7	8	9	10
# of dice	99										

Prediction #1 → What would you predict for the # of dice for trials 11 and 12? Make your prediction and then test it out.

Prediction #2 → Let’s say that our scenario becomes safe OVERALL when there are only 2 unsafe, radioactive molecules left, how many trials of our experiment would this take?

Experimental Results from IM2 2017/2018 class:

Trial #	Group Name/#	taymour and soc	Youssef, Jana	Easton, Jackson	Jae Hoon and O	Mehdi and Ahmed	Alice, Sakshi	Liam
0	99	99	99	99	99	99	99	99
1	86	83	89	83	90	77	86	84
2	65	72	73	74	80	58	69	66
3	55	66	63	59	61	45	56	57
4	46	51	54	51	50	41	47	49
5	35	44	48	44	38	36	42	37
6	30	37	40	34	31	31	34	34
7	25	33	36	30	26	23	25	25
8	24	28	30	28	23	19	22	23
9	19	24	25	23	18	15	17	16
10	15	19	23	21	17	14	13	13



## (f) CAC Payment Options

Mr. Rutherford is offering Mr. Santowski & Mr. Smith new contract options for the New Year. Here are the terms of the contracts being offered:

OPTION A → Here is Mr. Smith's payment option: Get paid \$5,000 US per day for each day in the month of January.

OPTION B → Here is Mr. Santowski payment option:

1. Get paid 1 piastre on the first day of January.
2. But then on the 2<sup>nd</sup> of January, return the 1 piastre and get paid double yesterday's wage, so get 2 piastres for having worked 2 days.
3. Now, on the 3<sup>rd</sup> of January, return the 2 piastres and get paid double yesterday's wage of 2 piastres, making it a total of 4 piastres pay for these three days.
4. Alas, on the 4<sup>th</sup> of January, return the 4 piastres and get paid double yesterday's wage of these 4 piastres, making it a total of 8 piastres pay for these four days.
5. Oh, woe is me. On the 5<sup>th</sup> of January, I return the 8 piastres, but get paid double yesterday's wage of these 8 piastres, making it a total of 16 piastres pay for these five days.
  - a. Which option would you choose and why?
  - b. Are the salaries ever equal? If so when? If not why not?
  - c. How much does each Math teacher get paid by the end of January? Convert to a common currency & show your work.

**PART 2 – Skills PRACTICE**

From the [Nelson 9 textbook, Chap 2.2](#) → starting on page 90, Q6,7,8,9,12

From the [Nelson 9 textbook, Chap 2.3](#) → starting on page 96, Q3,4,6,8