(A) Lesson Context

BIG PICTURE of this UNIT:	• mastery with algebraic skills to be used in our work with co-ordinate geometry (midpoint, length, slope) • understanding various geometric properties of quadrilaterals & triangles • how do you really "prove" that something is "true"?

(B) Lesson Objectives:

a. Use dynamic geometry programs (geogebra) to verifying properties of triangles and quadrilaterals and circles
b. Use dynamic geometry programs (geogebra) as a tool to decide on what needs to be proven and how to then plan an algebraic approach to verify the property in question
c. Use algebraic methods to verifying properties of triangles and quadrilaterals and circles

PART 1 – Skills REVIEW/EXPLORATION

1. Show that any mid-segment constructed from any two sides of the triangle with vertices at $P(-7,9)$, $Q(9,11)$, and $S(1,-11)$ is parallel to the third side. Key Steps to be demonstrated:

   a. Set up the diagram on Geogebra
   b. Research unknown concepts (what is a midsegment?)
   c. Use Geogebra to generate "relevant information"
   d. We will use this "relevant info" to help us to plan a strategy for "showing" what we are required to show ➔ we will try to use analytical geometry in this step
   e. Organize & present an solution

2. Using the same triangle with vertices at $P(-7,9)$, $Q(9,11)$, and $S(1,-11)$, use algebraic methods to determine its area. Key Steps to be demonstrated:

   a. Set up the diagram on Geogebra
   b. Research unknown concepts (different ways to find a triangle's area? What is an "altitude"?)
   c. Use Geogebra to generate "relevant information"
   d. We will use this "relevant info" to help us to plan a strategy for "showing" what we are required to show ➔ we will try to use analytical geometry in this step
   e. Organize & present an solution

3. Given the circle defined by $x^2 + y^2 = 125$, show that $A(10,5)$ and $B(-11,2)$ are on the circle and then secondly, show that the perpendicular bisector of CHORD AB goes through the center of the circle. Key Steps to be demonstrated:

   a. Set up the diagram on Geogebra
   b. Research unknown concepts (different ways to find a triangle's area? What is an "altitude"?)
   c. Use Geogebra to generate "relevant information"
   d. We will use this "relevant info" to help us to plan a strategy for "showing" what we are required to show ➔ we will try to use analytical geometry in this step
   e. Organize & present an solution

4. Show that the diagonals of the quadrilateral with vertices at $A(-6,4)$, $B(-2,6)$, $C(1,0)$ and $D(-3,-2)$ are equal in length. Make a conjecture about the type of quadrilateral. Key Steps to be demonstrated:

   a. Set up the diagram on Geogebra
   b. Research unknown concepts (different ways to find a triangle's area? What is an "altitude"?)
   c. Use Geogebra to generate "relevant information"
   d. We will use this "relevant info" to help us to plan a strategy for "showing" what we are required to show ➔ we will try to use analytical geometry in this step
   e. Organize & present an solution

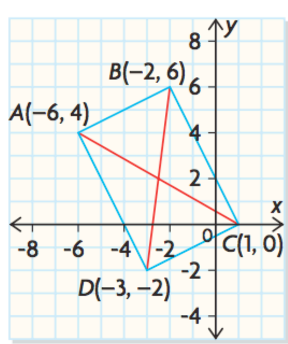
PART 2 – Skills PRACTICE/Applications & GEOMETRY Contexts

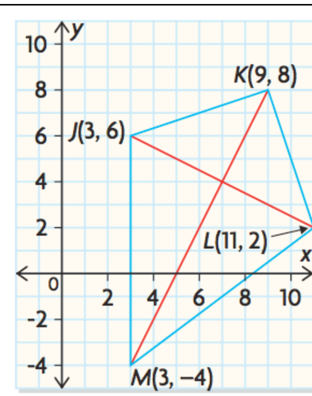
CHECK Your Understanding

1. Show that the diagonals of quadrilateral *ABCD* at the right are equal in length.

2. Show that the diagonals of quadrilateral *JKLM* at the far right are perpendicular.

3. $\triangle PQR$ has vertices at $P(-2, 1)$, $Q(1, 5)$, and $R(5, 2)$. Show that the median from vertex Q is the perpendicular bisector of PR.





4. A rectangle has vertices at $J(10, 0)$, $K(-8, 6)$, $L(-12, -6)$, and $M(6, -12)$. Show that the diagonals bisect each other.

5. A rectangle has vertices at $A(-6, 5)$, $B(12, -1)$, $C(8, -13)$, and
K $D(-10, -7)$. Show that the diagonals are the same length.

8. A triangle has vertices at $D(-5, 4)$, $E(1, 8)$, and $F(-1, -2)$. Show that the height from D is also the median from D.

9. Show that the midsegments of a quadrilateral with vertices at $P(-2, -2)$, $Q(0, 4)$, $R(6, 3)$, and $S(8, -1)$ form a rhombus.

10. Show that the midsegments of a rhombus with vertices at $R(-5, 2)$, $S(-1, 3)$, $T(-2, -1)$, and $U(-6, -2)$ form a rectangle.

11. Show that the diagonals of the rhombus in question 10 are perpendicular and bisect each other.

12. Show that the midsegments of a square with vertices at $A(2, -12)$, $B(-10, -8)$, $C(-6, 4)$, and $D(6, 0)$ form a square.

13. **a)** Show that points $A(-4, 3)$ and $B(3, -4)$ lie on $x^2 + y^2 = 25$.
b) Show that the perpendicular bisector of chord AB passes through the centre of the circle.

14. A trapezoid has vertices at $A(1, 2)$, $B(-2, 1)$, $C(-4, -2)$, and $D(2, 0)$.
A **a)** Show that the line segment joining the midpoints of BC and AD is parallel to both AB and DC.
b) Show that the length of this line segment is half the sum of the lengths of the parallel sides.

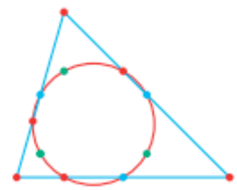
15. $\triangle ABC$ has vertices at $A(3, 4)$, $B(-2, 0)$, and $C(5, 0)$. Prove that the
T area of the triangle formed by joining the midpoints of $\triangle ABC$ is one-quarter the area of $\triangle ABC$.

16. Naomi claims that the midpoint of the hypotenuse of a right triangle is the same distance from each vertex of the triangle. Create a flow chart that summarizes the steps you would take to verify this property.

Higher Level Extension Work

Curious Math

The Nine-Point Circle



A circle that passes through nine different points can be constructed in every triangle. These points can always be determined using the same strategy.

1. On a grid, draw a triangle with vertices at $P(1, 11)$, $Q(9, 8)$, and $R(1, 2)$. Determine the midpoints of the sides, and mark each midpoint with a blue dot.
2. Determine the equation of each altitude. Then determine the coordinates of the point where the altitude meets the side that is opposite each vertex. Mark these points on your diagram in red.
3. Determine the coordinates of the orthocentre (the point where all three altitudes intersect). Then, for each altitude, determine the coordinates of the midpoint of the line segment that joins the orthocentre to the vertex. Mark these midpoints on your diagram in green.
4. Determine the coordinates of the circumcentre (the point where all three perpendicular bisectors intersect). Determine the midpoint of the line segment that joins the circumcentre to the orthocentre. Mark this midpoint with the letter N. This is the centre of your nine-point circle. Draw the circle.
5. Identify how each of the points that lie on the nine-point circle can be determined for any triangle.