(A) Lesson Context

BIG PICTURE of this UNIT:	• How do I determine the measure of angles in geometric shapes, without direct measurement? • How do I solve for sides or angles in right triangles? • How do I model real world scenarios using right triangles?

(B) **Lesson Objectives:**

a. Given a linear function in the for of $y = mx$, use geogebra to explore the relationship between slopes of lines and the angle of the lines.
b. Given a right triangle, use geogebra to explore the relationship between the ratios of triangle sides and the angles in the given triangle.

PART 1 – Skills REVIEW/EXPLORTION

Exploration #1 – RELATIONSHIPS – The relationship between the slope of a line and the angle it makes with the positive x axis.

1. Draw the line $y = mx$ in geogebra and add a slider for m.
2. Add the point (0,0) into your construction ➔ this will now be Point A
3. Add a point to the line (Point on Object tool) ➔ this will now be Point B
4. Construct a line perpendicular to the x-axis that goes through your point.
5. Construct the intersection point between the perpendicular line and the x-axis ➔ Point C
6. Construct triangle ABC (DON'T use the polygon tool ➔ use line segments)

7. (IF you are TOTALLY lost ➔ open the Geogebra Interactive I've created ➔ https://ggbm.at/warMhdcg)

Now that our construction of our right triangle and our line are complete, change the sliders (m as well as Point B) and observe what happens. Let's continue playing ……

8. Change the slope to $m = 1$ (so the equation of our line is now $y = 1x$) and have Geogebra measure angle CAB (the one at the origin) ➔ you should see the angle measure as 45°.
9. Recall that the slope ratio is calculated as rise/run, so have Geogebra measure the length of BC (rise) as well as the length of AC (run)
10. Now re-position Point B (the one on the line we started with) ➔ what happens to the slope ratio and what happens to the angle of the line? Does the location of Point B matter?

To start collecting data, use this Google sheet and we can enter our first "data point" of slope ratio of 1 and an angle of 45°➔ https://docs.google.com/a/cacegypt.org/spreadsheets/d/1eK_LI8dYiilT43V9LknTP9lv6a6lmW-KTxn74QMEw8s/edit?usp=sharing

11. Continue to play (change m to any value where $m \geq 0$) and thereby add more data (slope ratio and the line's angle) and see what happens to our graph of this relationship between slope ratio and angle of the line.

12. CONGRATULATIONS!!! We have just INVENTED a function, one that allows us to input a slope ratio and get an angle of a line as an output!!!!

13. How would work with this function if we knew the angle of the line (now the input) and wanted the slope ratio (now the output)?

BIG PICTURE KEY POINT ➔ We have used GEOMETRY to investigate a relationship (which we did by creating a DATA SET) and we can now analyze with a FUNCTION

Exploration #2 – RATIOS – The relationship between the angle of a triangle and the various ratios of its sides

Let's make another construction so that we can continue to explore relationships, now focusing in on RATIOS and ANGLES

1. Plot the point (0,0) ➔ this will now be Point A
2. Construct the circle $x^2 + y^2 = r^2$. Make sure you and your partner have different radii. Zoom in/out and center the circle.
3. Add a point on the circle ➔ this will now be point B
4. Construct a line perpendicular to the x-axis that goes through your point.
5. Construct the intersection point between the perpendicular line and the x-axis ➔ Point C
6. Construct triangle ABC (DON'T use the polygon tool ➔ use line segments)
7. Now "hide" the perpendicular line
8. (IF you are TOTALLY lost ➔ open the Geogebra Interactive I've created ➔ https://ggbm.at/TXf2k8y6

Now that our construction of our circle and our right triangle, change the sliders (for Point B) and observe what happens. Let's continue playing ……

9. Add to this data table ….. (but to keep it simple for now, let's just stay in the first quadrant (i.e. we will work with angles between $0° \leq \theta \leq 90°$ ➔ so the idea of DOMAIN again!!!)

Angle	Adjacent side	Opposite Side	Hypotenuse	Slope Ratio of $\frac{\text{opposite}}{\text{adjacent}}$	Ratio of $\frac{\text{opposite}}{\text{hypotenuse}}$	Ratio of $\frac{\text{adjacent}}{\text{hypotenuse}}$
$\theta = 45°$	0.71	0.71	1	$\frac{0.71}{0.71} = 1$	$\frac{0.71}{1} = 0.71$	$\frac{0.71}{1} = 0.71$
Etc …..						

To start collecting data, use this Google sheet and we can enter our first "data point" of an angle of 45° and a ratio (Opp/Hyp) of 0.71➔ https://docs.google.com/a/cacegypt.org/spreadsheets/d/1eK_LI8dYiilT43V9LknTP9lv6a6lmW-KTxn74QMEw8s/edit?usp=sharing

10. Continue to play (change θ to any value $0° \leq \theta \leq 90°$) and thereby add more data (angles and ratios) and see what happens to our graphs of these relationships between angles and ratio of sides in triangles (NOTE the tabs on the bottom of the data sheets!!)

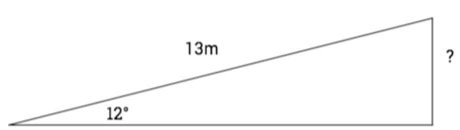
11. CONGRATULATIONS!!! We have just INVENTED two more functions, ones that allows us to input an angle and get ratio of two specific sides as an output!!!!

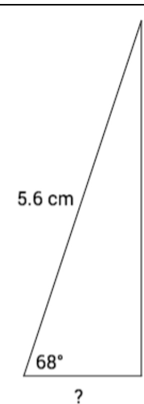
PART 2 – Skills PRACTICE

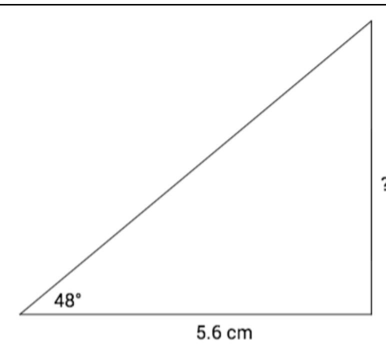
Now Mr. Rawlings and Mr. Santowski have gone through and measured a lot of triangles for angles between 1^o and 89^o .

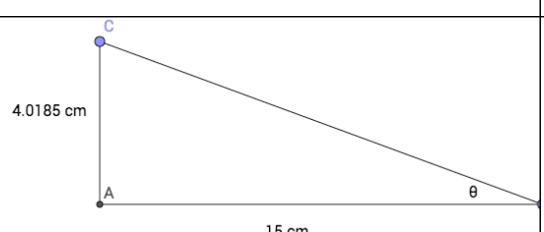
Angle θ	Opp/Hyp	Adj/Hyp	Opp/Adj		Angle θ	Opp/Hyp	Adj/Hyp	Opp/Adj
$\theta = 7^o$	.1219	.9925	.1228		$\theta = 48^o$	.7431	.6691	1.1101
$\theta = 12^o$	.2079	.9781	.2126		$\theta = 50^o$	.7660	.6428	1.1918
$\theta = 15^o$	.2588	.9695	.2679		$\theta = 52^o$	.7880	.6157	1.2799
$\theta = 21^o$	.3584	.9336	.3839		$\theta = 68^o$	.9272	.3746	2.4751
$\theta = 43^o$	.6820	.7313	.9325		$\theta = 89^o$	.9998	.0176	57.29
$\theta = 45^o$	.7071	.7071	1.0000					

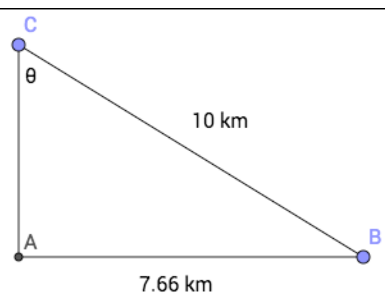
… This took a while. We found the same ratios that you did and here were our findings. See if you can use this table to help you find the missing lengths in the triangles given. Please explain your reasoning, show your work… etc.

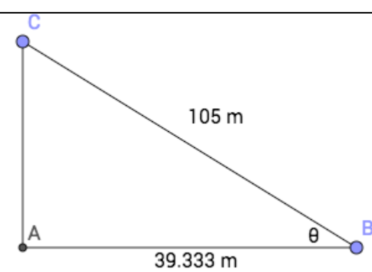












Angles & Ratios DATA TABLE

Angle	Adjacent side	Opposite Side	Hypotenuse	Slope Ratio of $\frac{\text{opposite}}{\text{adjacent}}$	Ratio of $\frac{\text{opposite}}{\text{hypotenuse}}$	Ratio of $\frac{\text{adjacent}}{\text{hypotenuse}}$
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