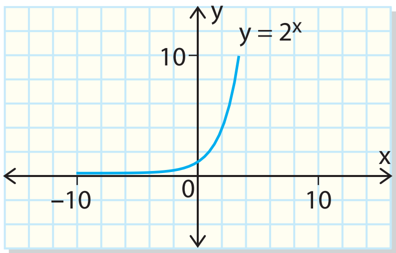
4 Chapter Review

FREQUENTLY ASKED Questions

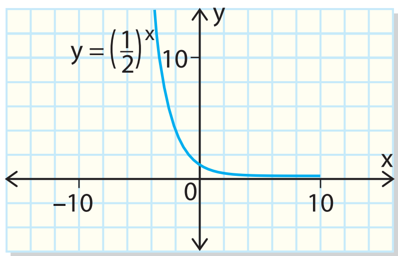
Q: How can you identify an exponential function from
- its equation?
- its graph?
- a table of values?

A: The exponential function has the form $f(x) = b^x$, where the variable is an exponent.

The shape of its graph depends upon the parameter b.



If $b > 1$, then the curve increases as x increases.



If $0 < b < 1$, then the curve decreases as x increases.

In each case, the function has the x-axis (the line $y = 0$) as its horizontal asymptote.

A differences table for an exponential function shows that the differences are never constant, as they are for linear and quadratic functions. They are related by a multiplication pattern.

x	$y = 3^x$	First Differences	Second Differences
0	1		
		2	
1	3		4
		6	
2	9		12
		18	
3	27		36
		54	
4	81		108
		162	
5	243		324
		486	
6	729		

Study *Aid*
- See Lesson 4.5.
- Try Chapter Review Questions 9 and 10.

Study Aid

- See Lesson 4.6, Examples 1, 2, and 3.
- Try Chapter Review Questions 11 and 12.

Q: **How can transformations help in drawing the graphs of exponential functions?**

A: Functions of the form $g(x) = af(k(x - d)) + c$ can be graphed by applying the appropriate transformations to the key points and asymptotes of the parent function $f(x) = b^x$, following an appropriate order—often, stretches and compressions, then reflections, and finally translations.

In functions of the form $g(x) = ab^{k(x-d)} + c$, the constants a, k, d, and c change the location or shape of the graph of $f(x)$. The shape of the graph of $g(x)$ depends on the value of the base of the function, $f(x) = b^x$.

- a represents the vertical stretch or compression factor. If $a < 0$, then the function has also been reflected in the x-axis.
- k represents the horizontal stretch or compression factor. If $k < 0$, then the function has also been reflected in the y-axis.
- c represents the number of units of vertical translation up or down.
- d represents the number of units of horizontal translation right or left.

Study Aid

- See Lesson 4.7, Examples 1, 2, 3 and 4.
- Try Chapter Review Questions 13 to 17.

Q: **How can exponential functions model growth and decay? How can you use them to solve problems?**

A: Exponential functions can be used to model phenomena exhibiting repeated multiplication of the same factor.

Each formula is modelled after the exponential function

$$y = ab^x$$

The amount in the future

The present value or initial amount

The growth/decay factor: if growth, then $b > 1$; if decay, then $0 < b < 1$

The number of periods of growth or decay

When solving problems, list these four elements of the equation and fill in the data as you read the problem. This will help you organize the information and create the equation you require to solve the problem.

Here are some examples:

Growth	Decay
Cell division (doubling bacteria, yeast cells, etc.): $P(t) = P_0(2)^{\frac{t}{D}}$	Radioactivity or half-life: $N(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{H}}$
Population growth: $P(n) = P_0(1 + r)^n$	Depreciation of assets: $V(n) = V_0(1 - r)^n$
Growth in money: $A(n) = P(1 + i)^n$	Light intensity in water: $V(n) = 100(1 - r)^n$

PRACTICE Questions

Lesson 4.2

1. a) If $x > 1$, which is greater, x^{-2} or x^2? Why?

b) Are there values of x that make the statement $x^{-2} > x^2$ true? Explain.

2. Write each as a single power. Then evaluate. Express answers in rational form.

a) $(-7)^3(-7)^{-4}$

b) $\dfrac{(-2)^8}{(-2)^3}$

c) $\dfrac{(5)^{-3}(5)^6}{5^3}$

d) $\dfrac{4^{-10}(4^{-3})^6}{(4^{-4})^8}$

e) $(11)^9\left(\dfrac{1}{11}\right)^7$

f) $\left(\dfrac{(-3)^7(-3)^4}{(-3^4)^3}\right)^{-3}$

Lesson 4.3

3. Express each radical in exponential form and each power in radical form.

a) $\sqrt[3]{x^7}$

b) $y^{\frac{8}{5}}$

c) $(\sqrt{p})^{11}$

d) $m^{1.25}$

4. Evaluate. Express answers in rational form.

a) $\left(\dfrac{2}{5}\right)^{-3}$

b) $\left(\dfrac{16}{225}\right)^{-0.5}$

c) $\dfrac{(81)^{-0.25}}{\sqrt[3]{-125}}$

d) $(\sqrt[3]{-27})^4$

e) $(\sqrt[5]{-32})(\sqrt[6]{64})^5$

f) $\sqrt[6]{((-2)^3)^2}$

5. Simplify. Write with only positive exponents.

a) $a^{\frac{3}{2}}(a^{-\frac{3}{2}})$

b) $\dfrac{b^{0.8}}{b^{-0.2}}$

c) $\dfrac{c\left(c^{\frac{5}{6}}\right)}{c^2}$

d) $\dfrac{d^{-5}d^{\frac{11}{2}}}{(d^{-3})^2}$

e) $((e^{-2})^{\frac{7}{2}})^{-2}$

f) $((f^{-\frac{1}{6}})^{\frac{6}{5}})^{-1}$

6. Explain why $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$, for $a > 0$ and $b > 0$.

Lesson 4.4

7. Evaluate each expression for the given values. Express answers in rational form.

a) $(5x)^2(2x)^3$; $x = -2$

b) $\dfrac{8m^{-5}}{(2m)^{-3}}$; $m = 4$

c) $\dfrac{2w(3w^{-2})}{(2w)^2}$; $w = -3$

d) $\dfrac{(9y)^2}{(3y^{-1})^3}$; $y = -2$

e) $(6(x^{-4})^3)^{-1}$; $x = -2$

f) $\dfrac{(-2x^{-2})^3(6x)^2}{2(-3x^{-1})^3}$; $x = \dfrac{1}{2}$

8. Simplify. Write each expression using only positive exponents. All variables are positive.

a) $\sqrt[3]{27x^3y^9}$

b) $\sqrt{\dfrac{a^6b^5}{a^8b^3}}$

c) $\dfrac{m^{\frac{3}{2}}n^{-2}}{m^{\frac{7}{2}}n^{-\frac{3}{2}}}$

d) $\dfrac{\sqrt[4]{x^{-16}(x^6)^{-6}}}{(x^4)^{-\frac{11}{2}}}$

e) $((-x^{0.5})^3)^{-1.2}$

f) $\dfrac{\sqrt{x^6(y^3)^{-2}}}{(x^3y)^{-2}}$

Lesson 4.5

9. Identify the type of function (linear, quadratic, or exponential) for each table of values.

a)

x	*y*
−5	−38
0	−3
5	42
10	97
15	162
20	237

b)

x	*y*
0	−45
2	−15
4	15
6	45
8	75
10	105

c)

x	y
1	13
2	43
3	163
4	643
5	2 563
6	10 243

e)

x	y
−2	2000
−1	1000
0	500
1	250
2	125
3	62.5

d)

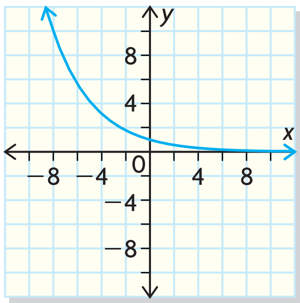
x	y
−2	40
−1	20
0	10
1	5
2	2.5
3	1.25

f)

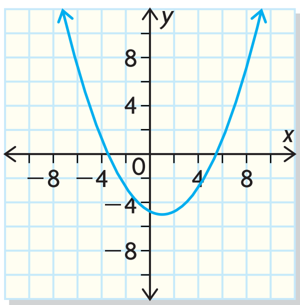
x	y
0.2	−10.8
0.4	−9.6
0.6	−7.2
0.8	−2.4
1	7.2
1.2	26.4

10. Identify each type of function (linear, quadratic, or exponential) from its graph.

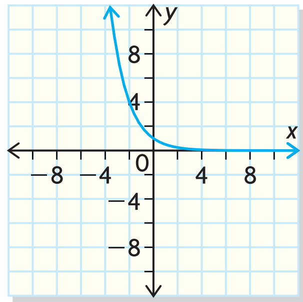
a)



b)



c)



Lesson 4.6

11. For each exponential function, state the base function, $y = b^x$. Then state the transformations that map the base function onto the given function. Use transformations to sketch each graph.

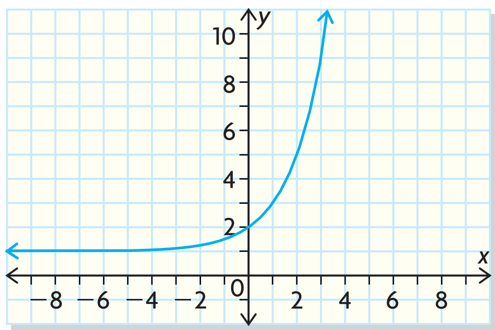
a) $y = \left(\frac{1}{2}\right)^{\frac{x}{2}} - 3$

b) $y = \frac{1}{4}(2)^{-x} + 1$

c) $y = -2(3)^{2x+4}$

d) $y = \frac{-1}{10}(5)^{3x-9} + 10$

12. The exponential function shown has been reflected in the y-axis and translated vertically. State its y-intercept, its asymptote, and a possible equation for it.



Lesson 4.7

13. Complete the table.

	Function	Exponential Growth or Decay?	Initial Value (y-intercept)	Growth or Decay Rate
a)	$V(t) = 100(1.08)^t$			
b)	$P(n) = 32(0.95)^n$			
c)	$A(x) = 5(3)^x$			
d)	$Q(n) = 600\left(\frac{5}{8}\right)^n$			

14. A hot cup of coffee cools according to the equation

$$T(t) = 69\left(\frac{1}{2}\right)^{\frac{t}{30}} + 21$$

where T is the temperature in degrees Celsius and t is the time in minutes.

a) Which part of the equation indicates that this is an example of exponential decay?
b) What was the initial temperature of the coffee?
c) Use your knowledge of transformations to sketch the graph of this function.
d) Determine the temperature of the coffee, to the nearest degree, after 48 min.
e) Explain how the equation would change if the coffee cooled faster.
f) Explain how the graph would change if the coffee cooled faster.

15. The value of a car after it is purchased depreciates according to the formula

$$V(n) = 28\,000(0.875)^n$$

where $V(n)$ is the car's value in the nth year since it was purchased.

a) What is the purchase price of the car?
b) What is the annual rate of depreciation?
c) What is the car's value at the end of 3 years?
d) What is its value at the end of 30 months?
e) How much value does the car lose in its first year?
f) How much value does it lose in its fifth year?

16. Write the equation that models each situation. In each case, describe each part of your equation.
a) the percent of a pond covered by water lilies if they cover one-third of a pond now and each week they increase their coverage by 10%
b) the amount remaining of the radioactive isotope U_{238} if it has a half-life of 4.5×10^9 years
c) the intensity of light if each gel used to change the colour of a spotlight reduces the intensity of the light by 4%

17. The population of a city is growing at an average rate of 3% per year. In 1990, the population was 45 000.
a) Write an equation that models the growth of the city. Explain what each part of the equation represents.
b) Use your equation to determine the population of the city in 2007.
c) Determine the year during which the population will have doubled.
d) Suppose the population took only 10 years to double. What growth rate would be required for this to have happened?