3.5 Quadratic Models Using Factored Form

GOAL

Determine the equation of a quadratic model using the factored form of a quadratic relation.

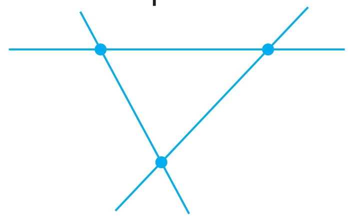
YOU WILL NEED

- graphing calculator
- grid paper
- ruler

INVESTIGATE the Math

You can draw one straight line through any pair of points. If you have three points you can draw a maximum of three lines. The maximum number of lines possible occurs when the points do not lie on the same line.

3 points



? What is the maximum number of lines you can draw using 100 points?

A. Can you answer the question directly using a diagram? Explain.

B. Since two points are needed to draw a line, using zero and one point results in zero lines. Copy and complete the rest of the table by drawing each number of points and determining the maximum number of lines that can be drawn through pairs of points.

Number of Points, *x*	0	1	2	3	4	5	6
Maximum Number of Lines, *y*	0	0					

C. Use your data to create a scatter plot with an appropriate scale.

D. What shape best describes your graph? Draw a **curve of good fit**.

curve of good fit
a curve that approximates, or is close to, the distribution of points in a scatter plot

E. Carry out appropriate calculations to determine whether the curve you drew for part D is approximately linear, approximately quadratic, or some other type.

F. What are the zeros of your curve? Use the zeros to write an equation for the relation in factored form: $y = a(x - r)(x - s)$.

G. Use one of the ordered pairs in your table (excluding the zeros) to calculate the value of a. Write an equation for the relation in both factored form and standard form.

H. Use a graphing calculator and **quadratic regression** to determine the equation of this quadratic relation model.

quadratic regression
a process that fits the second degree relation $y = ax^2 + bx + c$ to the data

curve of best fit

the curve that best describes the distribution of points in a scatter plot, usually found using a process called regression

Tech Support

For help using a TI-83/84 graphing calculator to determine the equation of a curve of best fit using quadratic regression, see Appendix B-10. If you are using a TI-*n*spire, see Appendix B-46.

I. How does your equation compare with the graphing calculator's **curve of best fit** equation?

J. Use your equation to predict the number of lines that can be drawn using 100 points.

Reflecting

K. How does the factored form of a quadratic relation help you determine the equation of a curve of good fit when it has two zeros?

L. How would the equation change if the data were quadratic and the curve of good fit had only one zero?

M. If a curve of good fit for a set of data had no zeros, could the factored form be used to determine its equation? Explain.

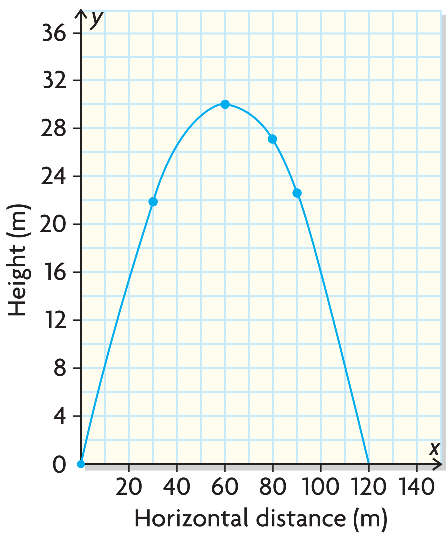
APPLY the Math

EXAMPLE 1 Connecting the zeros and factored form to an equation that models data

Data from the flight of a golf ball are given in this table. If the maximum height of the ball is 30.0 m, determine an equation for a curve of good fit.

Horizontal Distance (m)	0	30	60	80	90
Height (m)	0.0	22.0	30.0	27.0	22.5

Jill's Solution



I plotted the data. I drew a parabola as a curve of good fit because it seemed to be close to most of the data. The maximum height was 30.0 m, so the vertex of the parabola is located at (60, 30). The equation of the axis of symmetry is $x = 60$.

A zero occurs at (0, 0). Since a parabola is symmetric, I determined that another zero is located at (120, 0).

$$y = a(x - 0)(x - 120)$$

I wrote a general equation of the parabola in factored form. Because the zeros are 0 and 120, I knew that (x − 0) and (x − 120) are factors.

$$30 = a(60)(60 - 120)$$

$$\frac{30}{60(-60)} = a$$

$$\frac{1}{2(-60)} = a$$

$$-\frac{1}{120} = a$$

I substituted (60, 30) into the equation, since it is a point on the curve. Then I solved for *a*.

$$y = -\frac{1}{120}x(x - 120)$$

$$y = -\frac{1}{120}x^2 + x$$

I used the value of *a* to write the equation. Then I expanded the equation to write it in standard form.

When $x = 30$,

$$y = -\left(\frac{1}{120}\right)(30)(30 - 120)$$

$$y = -\left(\frac{1}{4}\right)(-90)$$

$$y = 22\frac{1}{2}$$

I checked the equation by substituting other values of *x* into it.

When $x = 80$,

$$y = -\left(\frac{1}{120}\right)(80)(80 - 120)$$

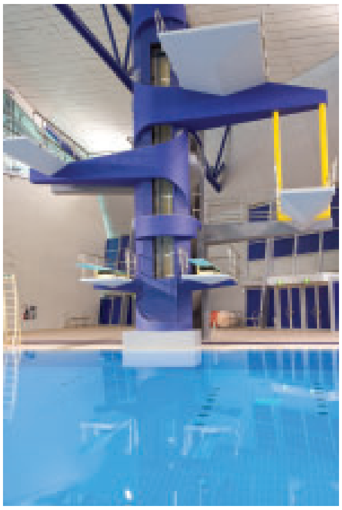
$$y = -\left(\frac{2}{3}\right)(-40)$$

$$y = 26\frac{2}{3}$$

The points $\left(30, 22\frac{1}{2}\right)$ and $\left(80, 26\frac{2}{3}\right)$ from the equation are close to the points (30, 22.0) and (80, 27.0) from the data.

The results for *y* were close to the values in the table, so the equation for the curve of good fit is reasonable.

An equation of good fit is $y = -\frac{1}{120}x^2 + x$.



EXAMPLE 2 | **Selecting an informal strategy** to determine an equation of a curve of good fit

A competitive diver does a handstand dive from a 10 m platform. This table of values shows the time in seconds and the height of the diver, relative to the surface of the water, in metres.

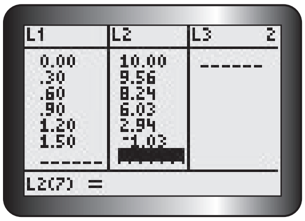
Time (s)	0	0.3	0.6	0.9	1.2	1.5
Height (m)	10.00	9.56	8.24	6.03	2.94	−1.03

Determine an equation that models the height of the diver above the surface of the water during the dive. Verify your result using quadratic regression.

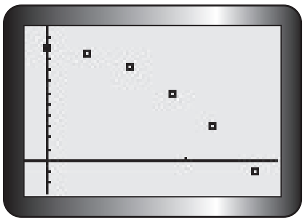
Madison's Solution

Tech Support

For help using a TI-83/84 graphing calculator to create a scatter plot, see Appendix B-10. If you are using a TI-*n*spire, see Appendix B-46.



I entered the data in the lists of a graphing calculator and created a scatter plot.



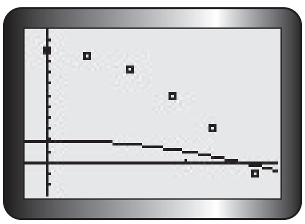
The points looked like they formed half of a parabola. I assumed that the diver was at the maximum height at the start of the dive. This meant the vertex was located at (0, 10). I estimated that one zero occurred at 1.4 and the other zero occurred at −1.4 since the *y*-axis is the axis of symmetry.

$y = a(x - 1.4)(x + 1.4)$

I wrote an equation of the parabola in factored form.

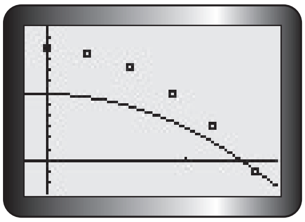
$y = -1(x - 1.4)(x + 1.4)$

I entered my equation into the equation editor using −1 as a guess for the value of *a*. I knew that *a* is negative since the parabola opens downward.



This graph is not a good fit.

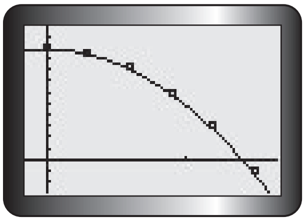
$y = -3(x - 1.4)(x + 1.4)$



I tried -3 as a value of *a* and graphed the relation again.

This graph is not a good fit either.

$y = -5(x - 1.4)(x + 1.4)$

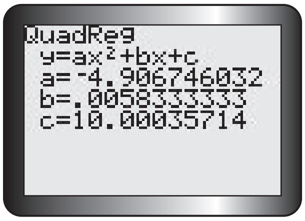


I tried -5 as a value of *a* and graphed the relation again.

This graph is a good fit that models the height of the diver above the surface of the water during the dive.

$$y = -5(x^2 + 1.4x - 1.4x - 1.96)$$
$$y = -5x^2 + 9.8$$

I expanded my equation to write it in standard form.



Then I used quadratic regression to determine the equation of the curve of best fit. My equation and the calculator's equation are very close.

EXAMPLE 3 Solving a problem using a model

Jeff and Tim are analyzing data collected from a motion detector following the launch of their model rocket.

Time (s)	0.0	1.0	2.0	3.0	4.0
Height (m)	0.0	16.0	20.0	15.5	0.0

a) Determine an equation for a curve of good fit.

b) Use the equation you determined for part a) to estimate the height of the rocket 0.5 s after it is launched.

Phil's Solution

a) A parabola might model this situation.

Since the height of the rocket increased and then decreased, I assumed that a quadratic model might be reasonable.

$$y = a(x - 0)(x - 4)$$
$$y = ax(x - 4)$$

I wrote a general equation of the relation in factored form. The zeros are 0 and 4, so $(x - 0)$ and $(x - 4)$ are factors.

$$20 = a(2)(2 - 4)$$
$$\frac{20}{(2)(-2)} = a$$
$$-5 = a$$

To determine the value of a, I substituted (2, 20) into the equation since it is a point on the curve.

$$y = -5x(x - 4)$$

I used the value of a to write the final equation of the curve of good fit.

When $x = 2$,
$y = -5(2)(2 - 4)$
$y = -5(2)(-2)$
$y = 20$

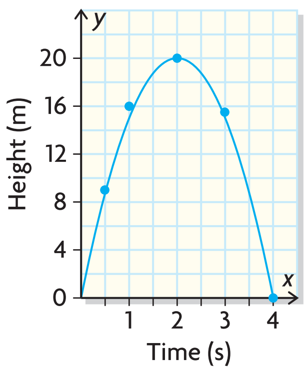
When $x = 3$,
$y = -5(3)(3 - 4)$
$y = -5(3)(-1)$
$y = 15$

I checked my equation by substituting other values of x into it. The results for y were close to the values in the table so my equation seems reasonable.

b) When $x = 0.5$,
$y = -5(0.5)(0.5 - 4)$
$y = -5(0.5)(-3.5)$
$y = 8.75$

I substituted 0.5 for x into the equation for the curve of good fit.

The height of the rocket after 0.5 s is approximately 8.8 m.



I checked the result by plotting the points and drawing the graph. The fit seems reasonable.

In Summary

Key Idea

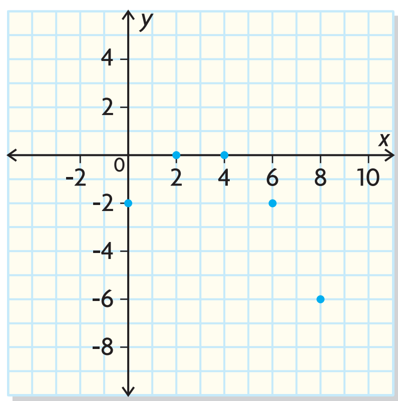
- If a curve of good fit for data with a parabolic pattern passes through the horizontal axis, then the factored form of the quadratic relation can be used to determine an algebraic model for the relationship.

Need to Know

- The estimated or actual x-intercepts, or zeros, of a curve of good fit represent the values of r and s in the factored form of the quadratic relation $y = a(x - r)(x - s)$.
- The value of a can be determined algebraically by substituting the coordinates of a point (other than a zero) that lies on or close to the curve of good fit into the equation and then solving for a.
- The value of a can be determined graphically by estimating the value of a and graphing the resulting parabola with graphing technology. By observing the graph, you can adjust your estimate of a and graph again until the parabola passes through or close to a large number of points in the scatter plot.
- Graphing technology can be used to determine an algebraic model for the curve of best fit. You can use quadratic regression when the data has a parabolic pattern.

CHECK Your Understanding

1. a) Use the graph at the right to determine an equation for a curve of good fit. Write the equation in factored and standard forms.



b) Use your equation to estimate the value of y when $x = 1$.

2. a) These data represent the path of a soccer ball as it flies through the air. Create a scatter plot, and then determine an equation for a quadratic curve of good fit.

Horizontal Distance (m)	0.0	1.0	2.0	3.0	4.0
Height (m)	1.0	1.6	1.9	1.6	1.0

b) Use your equation for part a) to estimate the height of the ball when its horizontal distance is 1.5 m.

3. a) Determine the equation of the quadratic curve of best fit for the data.

x	-1	0	1	2	3
y	-2.4	-3.6	-3.6	-2.3	0.1

b) Use your equation for part a) to estimate the value of y when $x = 3.2$.

PRACTISING

4. A parabola passes through the points $(-4, 10)$, $(-3, 0)$, $(-2, -6)$, $(-1, -8)$, $(0, -6)$, $(1, 0)$, and $(2, 10)$.
K
   a) Determine an equation for the parabola in factored form.
   b) Express your equation in standard form.
   c) Use a graphing calculator and quadratic regression to verify the accuracy of the equation you determined.

5. A water balloon was launched from a catapult. The table shows the data collected during the flight of the balloon using stop-motion photography.

Horizontal Distance (m)	0	6	12	18	24	30	36	42	48	54
Height (m)	0.0	11.6	20.4	26.4	29.5	29.7	27.1	21.6	13.3	2.1

   a) Use the data to create a scatter plot. Then draw a curve of good fit.
   b) Determine an equation for the curve you drew.
   c) Estimate the horizontal distance of the balloon when it reached its maximum height. Then use your equation to calculate its maximum height.
   d) Use your equation to determine the height of the balloon when its horizontal distance was 40 m.

6. An emergency flare was shot into the air from the top of a building. The table gives the height of the flare at different times during its flight.
A

Time (s)	0	1	2	3	4	5	6
Height (m)	60	75	80	75	60	35	0

   a) How tall is the building?
   b) Use the data in the table to create a scatter plot. Then draw a curve of good fit.
   c) Determine an equation for the curve you drew.
   d) Use your equation to determine the height of the flare at 2.5 s.

7. A hang-glider was launched from a platform on the top of the Niagara Escarpment. The data describe the first 13 s of the flight. The values for height are negative whenever the hang-glider was below the top of the escarpment.

Time (s)	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Height (m)	10.0	−0.8	−9.2	−15.2	−18.8	−20.0	−18.8	−15.2	−9.2	−0.8	10.0	23.2	38.8	56.8

a) Determine the height of the platform.
b) Determine an equation that models the height of the hang-glider over the 13 s period.
c) Determine the lowest height of the hang-glider and when it occurred.

8. The data in the table at the right represent the height of a golf ball at different times.

a) Create a scatter plot, and draw a curve of good fit.
b) Use your graph for part a) to approximate the zeros of the relation.
c) Determine an equation that models this situation.
d) Use your equation for part c) to estimate the maximum height of the ball.

Time (s)	Height (m)
0.0	0.000
0.5	10.175
1.0	17.900
1.5	23.175
2.0	26.000
2.5	26.375
3.0	24.300
3.5	19.775
4.0	12.800
4.5	3.375

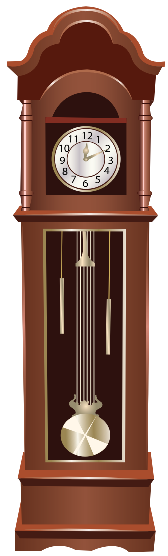
9. For a school experiment, Nichola recorded the height of a model rocket during its flight. The motion detector stopped working, however, during her experiment. The following data were collected before the malfunction.

Time (s)	0.0	1.0	2.0	3.0	4.0
Height (m)	2.00	19.5	27.0	24.5	12.0

a) The height–time relation is quadratic. Determine an equation for the height–time relation.
b) Use the equation you determined for part a) to estimate the height of the rocket at 3.8 s.
c) Determine the maximum height of the rocket. When did the rocket reach its maximum height?

10. A pendulum swings back and forth. The time taken to complete one back-and-forth swing is called the period.

Period (s)	0.5	1.0	1.5	2.0	2.5
Length of Pendulum (cm)	6.2	24.8	55.8	99.2	155.0



a) Can the data be represented by a quadratic relation? How do you know?
b) Use the data to draw a scatter plot. Then sketch a curve of good fit.
c) Assuming that your graph is a parabola with vertex (0, 0), determine an equation for your curve of good fit.
d) Estimate the period for a pendulum that is 80.0 cm long.
e) Estimate the length of a pendulum that has a period of 2.3 s.

11. Examine this square dot pattern.

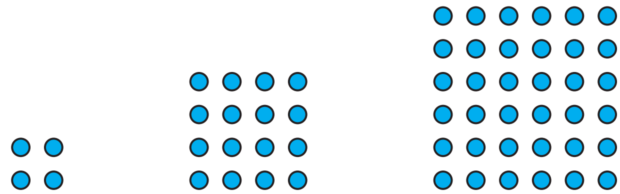


Diagram 1 Diagram 2 Diagram 3

How many dots are in the 20th diagram? Justify your answer.

12. Examine these three figures made of squares.

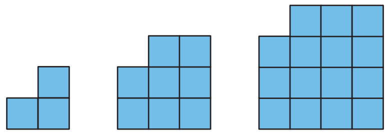


Figure 1 Figure 2 Figure 3

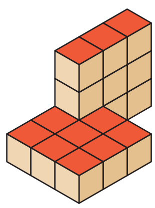
a) Create a table of values to compare the figure number, x, with the area, y. Draw figures 4 and 5 and add this data to your table.
b) Create a difference table to show that the relationship between the figure number and the area is quadratic.
c) Determine an equation for this relationship.
d) Using your difference table, work backwards to determine the zeros of this relationship.
e) Verify that the zeros you determined correspond to your equation.
f) What restriction must be placed on x to model this relationship accurately?

13. Can the factored form of a quadratic relation always be used to model a curve of good fit for data that appear to be quadratic? Explain.

14. Create a flow chart that summarizes the steps for determining the equation of a parabola of good fit using the factored form of a quadratic relation.

Extending

15. Examine this pattern of cube structures.



Model 1 Model 2 Model 3

a) Determine the number of cubes in the 15th model.
b) Which model in this pattern could you build using 1249 cubes?