Lesson 35 – Discrete Random Variables

SL2 Math - Santowski

Example #1

- We are collecting data in a probability game, wherein we are rolling 3 fair dice and determining the probabilities associated with the number of 6s that appear in our "game"
  - (a) complete a tree diagram
  - (b) prepare a relative frequency table
  - (c) prepare a relative frequency histogram
- Determine the probability that:
  - (a) you roll no 6's
  - (b) you roll at most one 6
  - (c) you roll exactly one 6, given that you rolled at most one 6)

Example #1

- So symbols and notations ➔ Let X represent the "variable" of the number of 6's rolled
- So $P(X = 1)$ is read as "the probability that one 6 is rolled" ➔ so $P(X=1) = 75/216$
- And likewise $P(X\leq 1) = P(X = 0) + P(X = 1) = 200/216$
- And then $P(X = 1|X < 1) = (75/216) \div (200/216)$

Probability Distribution

- A probability distribution could be understood as:
  1. a list or table showing the various outcomes and their associated probabilities
  2. a visual (a graph usually) relating the various outcomes and their associated probabilities
  3. an equation relating the various outcomes and their associated probabilities
- What is a "variable"? a numerically valued characteristic that will change (or vary), depending upon the outcome of the "probability experiment/event" we are performing

Introductory Example #2

- Suppose Egypt's national football (soccer that is) team is playing against USA's national team and the game is here in Cairo.
- To generate data in our "experiment," we will go into the stadium and select three fans randomly and identify whether the fan supports the Egyptian team (E) or the US team (U)
- Create a tree diagram to show the outcomes of our "probability experiment" and determine the probabilities associated with the different outcomes.

Introductory Example #2

- Suppose Egypt's national football (soccer that is) team is playing against USA's national team and the game is here in Cairo.
- To generate data in our "experiment," we will go into the stadium and select three fans randomly and identify whether the fan supports the Egyptian team (E) or the US team (U)
- The experiment gives us the following sample space:

$$\{(EEE),(EEU),(EUE),(UEE),(EUU),(UUE),(UEU),(UUU)\}$$

Introductory Example #2

- Now, we will let X represent the number of fans selected who are supporting Egypt
- Therefore, the possible values of X are 0,1,2, or 3
- Now let's work out the probability that X takes on the value of 0 ➔
- And then let's work out the probability that X takes on the value of 1 ➔
- And then let's work out the probability that X takes on the value of 2 ➔
- And then let's work out the probability that X takes on the value of 3 ➔
- So we are effectively working out the probability that X takes on the value of x (where x = 0,1,2,3) ➔ we shall denote this idea as P(X = x)
- To determine the probabilities, let's say the game is in Cairo so P(E) = 0.9 and let's further say that our probabilities are independent ➔ so therefore ……..

Introductory Example #2

- To determine the probabilities, let's say the game is in Cairo and P(E) = 0.9
- So to find P(X = 3) ➔ (i.e. all three fans we asked supported the Egyptian team:
- P(X = 3) = P(E) x P(E) x P(E) = 0.9 x 0.9 x 0.9= 0.729
- To find P(X = 2) ➔ that two of the three fans support Egypt: i.e ➔ EEU,EUE, UEE
- P(X = 2) = 0.9x0.9x0.1 + 0.9x0.1x0.9 + 0.1x0.9x0.9 = 0.243
- To find P(X = 1) ➔ that one of the three fans support Egypt: i.e ➔ EUU,UUE, UEU
- P(X = 1) = 0.9x0.1x0.1 + 0.1x0.1x0.9 + 0.1x0.9x0.1 = 0.027
- To find P(X = 0) ➔ that none of the three fans support Egypt: i.e ➔ UUU
- P(X = 0) = 0.1x0.1x0.1 = 0.001

Introductory Example #2

- Several concluding comments:
- We will say that the probabilities in our experiment "behave well" in that (i) the probabilities are greater than zero: P(X = x) > 0 and (ii) the probability of our sample space is 1: P(X=3) + P(X=2) + P(X=1) + P(X=0) = 1
- Because the values that X takes on are random, the variable X has a special name ➔ its called a **random variable**!

Random Variable - Definition

- What is a "**variable**"? ➔ a numerically valued characteristic that will change (or vary), depending upon the outcome of the "probability experiment/ event" we are performing
- A **random variable** is a numerical measure of the outcome from a probability experiment, so its value is determined by chance. Random variables are denoted using letters such as *X*.

Random Variables – Two Types

- A **discrete random variable** is a random variable that has values that has either a finite number of possible values or a countable number of possible values.
- A **continuous random variable** is a random variable that has an infinite number of possible values that is not countable.

Discrete & Continuous Data

- Discrete data is one in which all possible outcomes can be clearly identified and counted:
  - Die Roll: S = {1,2,3,4,5,6}
  - Number of Boys among 4 children: S = {0, 1, 2, 3, 4}
  - Number of baskets made on two free throws: S = {0, 1, 2}
- Data that can has a clearly finite number of values is known as discrete data.
- Non-discrete data, which is called called a continuous data, is one in which the outcomes are too numerous to identify every possible outcome:
  - Height of Human Beings: S = [0.00 inches, 100.00 inches] → It is impractical to specify every possible height from 0.00 to 100.00.
  - Muscle gain over 1 year of weight training: [0 pounds, 100+ pounds]

Discrete Random Variable Examples

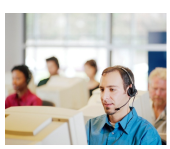
Experiment	Random Variable	Possible Values
Make 100 Sales Calls	# Sales	0, 1, 2, ..., 100
Inspect 70 Radios	# Defective	0, 1, 2, ..., 70
Answer 33 Questions	# Correct	0, 1, 2, ..., 33
Count Cars at Toll Between 11:00 & 1:00	# Cars Arriving	0, 1, 2, ..., ∞

Continuous Random Variable Examples

Experiment	Random Variable	Possible Values
Weigh 100 People	Weight	45.1, 78, ...
Measure Part Life	Hours	900, 875.9, ...
Amount spent on food	$ amount	54.12, 42, ...
Measure Time Between Arrivals	Inter-Arrival Time	0, 1.3, 2.78, ...

Two Types of Random Variables

- **Discrete random variables**
  - Number of sales
  - Number of calls
  - Shares of stock
  - People in line
  - Mistakes per page



- **Continuous random variables**
  - Length
  - Depth
  - Volume
  - Time
  - Weight

Notations used with DRV

- We use capital letter, like X, to denote the random variable and use small letter to list the possible values of the random variable.
- Example. A single dice is rolled, X represent the number of dots showing on the top face of the dice and the possible values of X are x=1,2,3,4,5,6.
- The statement P(X = x) signifies the probability that the random variable X takes on possible value of x

Probability Distributions for DRV

- The **probability distribution** of a discrete random variable is a graph, table or formula that specifies the probability associated with each possible outcome the random variable can assume.
  - $p(x) \geq 0$ for all values of x
  - $\Sigma p(x) = 1$

Discrete Probability Distribution Example

Experiment: Toss 2 coins. Count number of tails.

Probability Distribution

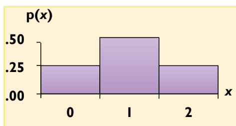
Values, x	Probabilities, $p(x)$
0	**1/4 = .25**
1	**2/4 = .50**
2	**1/4 = .25**

Visualizing Discrete Probability Distributions

Listing

{ (0, .25), (1, .50), (2, .25) }

Graph



Table

# Tails	f(x) Count	p(x)
0	1	.25
1	2	.50
2	1	.25

Formula (to be explained later)

$$p(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

Example #1 – Using a Formula

- A discrete random variable, X, has a probability distribution function given by $f(x) = P(X = x) = kx^2$ where $x = 1,2,3,4$
- (a) Prepare a probability distribution table
- (b) Hence or otherwise, determine the value of k.
- (c) Prepare a bar chart/histogram to represent f(x)
- (d) Calculate $P(X = 3)$
- (e) Calculate $P(X = 3 \text{ or } X = 4)$
- (f) Calculate $P(1 \leq X \leq 3)$

Example #2 – Given Probabilities

- The probabilities that a customer selects 1, 2, 3, 4, or 5 items at a convenience store are 0.32, 0.12, 0.23, 0.18, and 0.15 respectively.
  - (a) Construct a probability distribution (table) for the data and draw a probability distribution histogram.
  - (b) Find $P(X > 3.5)$
  - (c) Find $P(1.0 \leq X < 3.0)$
  - (d) Find $P(X < 5)$
  - (e) Calculate $P(X = 3)$
  - (f) Calculate $P(X = 3 \text{ or } X = 4)$
  - (g) How probable is it that a customer selects at least 2 items?

Example #3 – Contextual

- From past experience, a company has found that in carton of transistors, 92% contain no defective transistors, 3% contain one defective transistor, 3% contain two defective transistors, and 2% contain three defective transistors.
- (a) Construct a probability distribution.
- (b) Draw a probability distribution histogram.
- (c) Draw a cumulative probability distribution histogram
- (d) Calculate the mean, variance, and standard deviation for the defective transistors

Example #4 - Contextual

- There are 8 red socks and 6 blue socks in a drawer. A sock is taken out, its color noted and returned to the drawer. This procedure is repeated 4 times.
- Let R be the random variable "number of red socks taken."
- (a) Determine the probability distribution of R and represent it in a chart and a graph
- (b) The procedure is repeated, but this time each sock is NOT returned after it has been taken out.
- (c) The procedure is repeated in the DARK. How many socks must be taken out so that you have a matching pair?

Summary Measures

1. Expected Value (Mean of probability distribution)
   - Weighted average of all possible values
   - $\mu = E(x) = \Sigma x\, p(x)$
2. Variance
   - Weighted average of squared deviation about mean
   - $\sigma^2 = E[(x - \mu)^2] = \Sigma\, (x - \mu)^2\, p(x)$
3. Standard Deviation
   - $\sigma = \sqrt{\sigma^2}$

Example

3 A discrete random variable x can assume five possible values: 12, 13, 15, 18, and 20. Its probability distribution is shown below.

x	12	13	15	18	20
P(x)	0.14	0.11		0.26	0.23

a) What is P(15)?
b) What is the probability that x equals 12 or 20?
c) What is P($x \leq 18$)?
d) Find E(x).
e) Find V(x).(*)

Example

4 Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer. In a study to check the validity of such a claim, researchers chose different treatment centres and chose five of their patients at random. Here is the probability distribution of the number of successful treatments for groups of five:

x	0	1	2	3	4	5
P(x)	0.002	0.029	0.132	0.309	0.360	0.168

a) Find the probability that at least two patients would benefit from the treatment.
b) Find the probability that the majority of the group does not benefit from the treatment.
c) Find E(x) and interpret the result.
d) Show that $\sigma(x) = 1.02$.(*)
e) Graph P(x). Locate μ, $\mu \pm \sigma$ and $\mu \pm 2\sigma$ on the graph. Use the empirical rule to approximate the probability that x falls in this interval. Compare this with the actual probability.

Example

6 x has probability distribution as shown in the table.

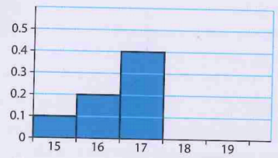
x	5	10	15	20	25
P(x)	$\frac{3}{20}$	$\frac{7}{30}$	k	$\frac{3}{10}$	$\frac{13}{60}$

a) Find the value of k.
b) Find P($x > 10$).
c) Find P($5 < x \leq 20$).
d) Find the expected value and the standard deviation.

Example

8 The probability distribution of students categorized by age that visit a certain movie house on weekends is given below. The probabilities for 18- and 19-year-olds are missing. We know that

$$P(x = 18) = 2P(x = 19)$$



a) Complete the histogram and describe the distribution.
b) Find the expected value and the variance.

Example

9 In a small town, a computer store sells laptops to the local residents. However, due to low demand, they like to keep their stock at a manageable level. The data they have indicate that the weekly demand for the laptops they sell follows the distribution given in the table below.

X: number of laptops bought	0	1	2	3	4	5
$P(X = x)$	0.10	0.40	0.20	0.15	0.10	0.05

a) Find the mean and standard deviation of this distribution.
b) Use the empirical rule to find the approximate number of laptops that is sold about 95% of the time.
c) Is it likely that 5 or more customers buy a laptop in any week?