Lesson 34 – Probability of Compound Events and Formulas

(A) Lesson Objectives

a. Review of Probability Rules & Terms
b. Work through compound events using Venn diagrams as our problem solving strategy
c. Introduce the concept of conditional probability through the use of Venn diagrams

(B) Review of Probability of Compound Events

a. Addition Rule of Probability
b. Multiplication Rule of Probability
c. Compliment of an Event
d. Symbols used
e. Conditional Probability
f. Mutually Exclusive Events
g. Independent Events
h. Dependent Events

(C) Examples

4. The events A and B are such that $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.2$.
Find
(a) $P(A \cup B)$ (b) $P(B')$ (c) $P(A' \cap B)$

5. The events A and B are such that $p(A) = 0.35$, $p(B) = 0.5$ and $p(A \cap B) = 0.15$.
Using a Venn diagram (where appropriate) find:
(a) $p(A')$ (b) $p(A \cup B)$ (c) $p(A \cup B')$.

6. The events A and B are such that $p(A) = 0.45$, $p(B) = 0.7$ and $p(A \cap B) = 0.20$.
Using a Venn diagram (where appropriate) find:
(a) $p(A \cup B)$ (b) $p(A' \cap B')$ (c) $p((A \cap B)')$.

A football team has a 70% chance of winning when it does not snow, but only a 40% chance of winning when it does snow. Suppose there is a 50% chance of snow. Find the probability that the team will win. Are the events of "snow" and "winning" dependent or independent? Explain how you know.

Events A and B are given such that $P(A) = \frac{3}{4}$ and $P(A \cup B) = \frac{4}{5}$ and $P(A \cap B) = \frac{3}{10}$. Find P(B).

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Micah has just graduated with a hospitality degree from VCC. She has applied for a job in Ontario at a four star hotel. The probability that she will get the job is 0.35. The probability that she will move to Ontario if she receives a job offer is 0.85. The probability that she will move to Ontario if she does not receive a job offer is 0.40. Create a tree diagram for all possible outcomes. What is the probability that she will be offered the job and not move to Ontario (to 4 decimal places)?

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11. A and B are two events such that $P(A) = p$, $P(B) = 2p$ and $P(A \cap B) = p^2$.

(a) Given that $P(A \cup B) = 0.4$, find p.

Use a Venn Diagram to help you find the following:

(b) $P(A' \cup B)$

(c) $P(A' \cap B')$.

Five percent of my students suffer from a terrible malady called Lazybrain(LB). A blood test detects LB accurately 90% of the time. Yusuke is told that his blood test is positive for LB. Yusuke hopes that this is a "false positive" and he actually doesn't have Lazybrain.

i. Draw the tree with all the probabilities. Indicate which branches are the *false positive, false negative, correct positive,* and *correct negative.*

ii. Find the probability that Yusuke is OK even though his blood test was positive.

Events A and B are given such that $P(A) = \frac{7}{10}$ and $P(A \cup B) = \frac{9}{10}$ and $P(A \cap B) = \frac{3}{10}$. Find

(a) P(B) (b) $P(B' \cap A)$ (c) $P(B \cap A')$ (d) $P(B' \cap A')$ (e) $P(B \mid A')$

The owner of a local yoga studio tells you that the probability of a visitor buying a membership is 65%. The probability that someone will buy a membership and sign up for a yoga class is 26%. The probability that someone will not sign up for a class given that they did not buy a membership is 75%. (a) What is the probability that a visitor to the studio will sign up for a class, given that they bought a membership? (b) What is the probability that someone does not purchase a membership and signs up for a class?

Suppose you throw a pair of fair 6-sided dice. One is white and the other is black. Let T = total showing on both dice, and B = number showing on the black die.

a) Find $P(T = 5 \mid B = 2)$ b) Find $P(B = 2 \mid T = 5)$

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1. Two events A and B are such that $p(A) = 0.6$, $p(B) = 0.4$ and $p(A \cap B) = 0.3$. Find the probability of the following events:

(a) $A \cup B$ (b) $A \mid B$ (c) $B \mid A$ (d) $A \mid B'$

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A small school has 24 boys graduating. Half of them are funny and 7 are good dancers. Eight of them are neither funny nor good dancers. One boy is selected at random. Translate the following into conditional probability notation, then find the probabilities.

i. Probability he is a good dancer given that he is funny.
ii. Probability he is funny given that he is a good dancer.
iii. If he is a good dancer, what is the probability he is not funny?
iv. If he is not a good dancer, what is the probability he is funny?

2. A and B are two events such that $p(A) = 0.3$, $p(B) = 0.5$ and $p(A \cup B) = 0.55$

Find the probability of the following events:

(a) $A|B$ (b) $B|A$ (c) $A|B'$ (d) $A'|B'$

Isabel goes to school by one of two routes, A or B. The probability of going by route A is 30%. If she goes by route A, the probability of being late is 5% and if she goes by route B, the probability of being late is 10%

i. Draw a tree diagram
ii. Find the probability that Isabel is late for school.
iii. Given that she is late for school, find the probability that she went to school using route A

9. Given that $p(A) = 0.6$, $p(B) = 0.7$ and that A and B are independent events.

Find the probability of the event

(a) $A \cup B$ (b) $A \cap B$ (c) $A|B'$ (d) $A' \cap B$

Amber, a college senior, interviews with Acme Corp. and Mills, Inc. The probability of receiving an offer from Acme is 0.35, from Mills is 0.48, and from both is 0.15. Find the probability of receiving an offer from either Acme Corp. or Mills, Inc., but not both.

Determine if events *A* and *B* are independent.

9) $P(A) = \frac{2}{5}$ $P(B) = \frac{1}{5}$ $P(A \text{ and } B) = \frac{2}{25}$

10) $P(A) = \frac{2}{5}$ $P(B) = \frac{1}{4}$ $P(A \text{ and } B) = \frac{1}{25}$

11) $P(A) = \frac{9}{20}$ $P(B) = \frac{1}{2}$ $P(A|B) = \frac{27}{50}$

12) $P(\text{not } A) = \frac{3}{4}$ $P(B) = \frac{3}{10}$ $P(A \text{ and } B) = \frac{3}{40}$

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Ten students are trying out for three positions on a coed soccer team. The students include four boys (Adam, Alex, Anthony and Arnold) and six girls (Abbey, Aurora, Agnes, Alice, Amanda and Anna). All the students have an equal chance of being selected for the team.

(a) How many different three-member teams can be formed? ________________

(b) Determine the probability that the team would include:

Three boys: ____

One boy and two girls: _______________________________________

At most one girl: ____

Adam, Anthony and Alice: ____________________________________

Agnes and two other students: ______________________________________

(c) How is the number of ways to select the team affected if the three openings on the team are specifically for positions of forward, midfield and defense? That is, with each selection, a student's name is attached to a specific position.

Determine if events *A* and *B* are independent.

9) $P(A) = \frac{2}{5}$ $P(B) = \frac{1}{5}$ $P(A \text{ and } B) = \frac{2}{25}$

10) $P(A) = \frac{2}{5}$ $P(B) = \frac{1}{4}$ $P(A \text{ and } B) = \frac{1}{25}$

11) $P(A) = \frac{9}{20}$ $P(B) = \frac{1}{2}$ $P(A|B) = \frac{27}{50}$

12) $P(\text{not } A) = \frac{3}{4}$ $P(B) = \frac{3}{10}$ $P(A \text{ and } B) = \frac{3}{40}$

Each member of a sports club plays at least one of soccer, rugby or tennis. The following is known: 43 members play tennis, 11 play tennis & rugby, 7 play tennis & soccer, 6 play soccer & rugby, 84 play rugby or tennis, 68 play soccer or rugby and 4 play all three sports.

(i) How many members does the club have?

(ii) Two members are chosen at random. How probable is it that both play rugby?

(iii) Two members are chosen at random. How probable is it that both play rugby, given that neither plays tennis?

Events A and B are independent. Find the missing probability.

13) $P(A) = \frac{1}{4}$ $P(B) = \frac{3}{5}$ $P(B|A) = ?$

14) $P(B) = \frac{9}{20}$ $P(A|B) = \frac{1}{5}$ $P(A) = ?$

15) $P(A) = \frac{3}{10}$ $P(B) = \frac{13}{20}$ $P(A \text{ and } B) = ?$

16) $P(B) = \frac{9}{20}$ $P(A \text{ and } B) = \frac{9}{100}$ $P(A) = ?$

Jar A has 4 red and 5 black candies. Jar B has 6 red and 2 black candies. A fair die is rolled and jar A is selected if a number divisible by 3 comes up, otherwise, Jar B is selected. One candy is drawn from the jar.

a) What is the probability you selected Jar A and got a red candy?
b) What is the probability you selected Jar B and got a red candy?
c) What is the probability you got a red candy?
d) Suppose a red candy is drawn, what is the probability it came from jar A?
e) What is the probability the candy was red, given that the candy came from jar A?
f) What is the probability Jar B was selected if a black candy is drawn?

There is a 40% chance of heavy snow and a 60% chance of light snow. If there is a heavy snow then there is a 80% chance that the school closes. If there is a light snow there is only a 30% chance that the school closes. Answer the following from your tree diagram:

a. P(school closes)
b. P(heavy snow and school remains open)
c. P(heavy snow given that the school closes)
d. P(light snow given that the school remains open)
e. P(if school closes then there is light snow)
f. P(if school remains open then there is heavy snow)

Participants in a study of a new medication received either medication A or a placebo. Make a tree diagram of the results of the study. Of all those who participated in the study 80% received medication A

- Of those who received medication A, 76% reported an improvement
- Of those who received the placebo, 62% reported no improvement

Then find (i) P(placebo and improvement), (ii) P(improvement), (iii) P(placebo was received given that improvement was noticed)