

**(A) Lesson Context**

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>• How can we visualize events and outcomes when considering compound events ?</li> <li>• How can we calculate probabilities when considering compound events ?</li> </ul>		
CONTEXT of this LESSON:	<p>Where we've been</p> <p>We've visualized compound events (tree diagram &amp; lists) and then calculated AND/OR probabilities for compound events &amp; mutually exclusive/inclusive events</p>	<p>Where we are</p> <p>When dealing with compound events, WHY do we &amp; HOW do we handle independent and dependent events?</p>	<p>Where we are heading</p> <p>Can we predict how likely is an event to occur? How can we use that knowledge?</p>

**(B) Lesson Objectives:**

- a. be able to distinguish between compound events that are either INDEPENDENT or INDEPENDENT
- b. be able to perform probability calculations once having identified the events as INDEPENDENT or DEPENDENT

**(C) Opening Discussion Questions**

- a. Two seniors, one from each government class are randomly selected to travel to Washington, D.C. Wes is in a class of 18 students and Maureen is in a class of 20 students. Find the probability that both Wes and Maureen will be selected.
  
- b. If there was only one government class, and Wes and Maureen were in that class of 38 students, what would be the probability that both Wes and Maureen would be selected as the two students to go to Washington? Is this still an example of independent events?
  
- c. A box contains 5 purple marbles, 3, green marbles, and 2 orange marbles. Two consecutive draws are made from the box **without replacement** of the first draw. Find the probability of each event, given the following conditions: (i) the marbles are NOT REPLACED after each draw and then secondly, (ii) the marbles ARE REPLACED after each draw

<u>(i) Not Replaced</u>	<u>(ii) Replaced</u>
<p>P(orange first, green second) =</p> <p>P(both marbles are purple) =</p> <p>P( the first marble is purple, and the second is ANY color EXCEPT purple) =</p>	<p>P(orange first, green second) =</p> <p>b. P(both marbles are purple) =</p> <p>c. P( the first marble is purple, and the second is ANY color EXCEPT purple) =</p>

**(D) Key Terms for Independent & Dependent COMPOUND EVENTS**

INDEPENDENT compound events – Two events are said to be **independent** if the result of the second event is not affected by the result of the first event.

DEPENDENT compound events – If the result of one event **IS** affected by the result of another event, the events are said to be **dependent**. **OR** another perspective on it is “because what happens **depends on** what happened before”

Examples: Are the events independent or dependent?

- a. Roll a die; toss a coin
- b. Take a marble out of a bag; take a second marble out of a bag.
- c. Choose a person from a group of 50 persons. Choose another person from the same group.
- d. Draw a card from the deck and put it back. Draw a card again from the same deck.
- e. You have a jar with 24 pieces of chocolate candy and 14 pieces of orange candy. We take one piece of candy at random from the jar, put it back, and then take a second piece of candy at random from the jar.
- f. Deni has a blue, red, and green tie. He also has a blue and green shirt. Deni chooses a random tie and shirt for work today.
- g. Amy plays card games. He picks a card at random. Then without putting the first card back, he picks a second card at random.
- h. Juan has 14 coins. He takes 3 of them at random, then he puts these back, and then pick 2 more coins at random.
- i. Brett has \$4,700 in his bank account. He withdraws \$1,200 from his bank account to pay for rent. Brett books a vacation 3-days later that costs \$4,000. He withdraws \$3,500 from his account and goes on a payment pay for the remainder.
- j. Canady has 10 handmade sheets. She takes 6 sheets at random. Then without putting these sheets back, she picks 2 sheets at random.
- k. Jeff has 3 children. His first 2 children are boys. His last child is a girl.

**(E) Simple Examples:**

- (a) Bag A contains 9 red marbles and 3 green marbles. Bag B contains 9 black marbles and 6 orange marbles. Find the probability of selecting one green marble from bag A and one black marble from bag B.
- (b) A coin is tossed and a die with numbers 1-6 is rolled. What is  $P(\text{heads and } 3)$ ?
- (c) Two cards are selected from a deck of cards numbered 1 – 10. Once a card is selected, it is not replaced. What is  $P(\text{two even numbers})$ ?
- (d) A club has 25 members, 20 boys and 5 girls. Two members are selected at random to serve as Executive Officers (i.e president and vice president.) What is the probability that both executive officers will be girls?
- (e) One marble is randomly drawn and then replaced from a jar containing two white marbles and one black marble. A second marble is drawn. What is the probability of drawing a white and then a black?
- (f) Maria rolls a pair of dice. What is the probability that she obtains a sum that is either a multiple of 3 OR a multiple of 4?
- (g) Events A and B are independent. The  $P(A) = 3/5$ , and  $P(\text{not } B) = 2/3$ . What is  $P(A \text{ and } B)$ ? What is  $P(A \text{ or } B)$ ?
- (h) Events A and B are independent. If  $P(A) = 0.21$ ,  $P(B) = 0.34$ , find  $P(A \text{ and } B) = ?$ . What is  $P(A \text{ or } B)$ ?
- (i) Events A and B are independent. If  $P(B) = 0.8$ ,  $P(A \text{ and } B) = 0.40$ , find  $P(A) = ?$ . What is  $P(A \text{ or } B)$ ?
- (j) In a survey at a football game, 50 of 75 male fans and 40 of 50 female fans said that they favor the new team mascot. If 1 male and 1 female are randomly selected, what is the probability that both favor the new mascot?
- (k) One bag contains 2 green marbles and 4 white marbles, and a second bag contains 3 green marbles and 1 white marble. If Trent randomly draws one marble from each bag, what is the probability that they are both green?
- (l) On a certain day the chance of rain is 80% in San Francisco and 30% in Sydney. Assume that the chance of rain in the two cities is independent.
- What is the probability that it will not rain in either city?
  - What is the probability that it will not rain in BOTH cities?
- (m) A math teacher is randomly distributing 15 rulers with centimeter labels and 10 rulers without centimeter labels. What is the probability that the first ruler she hands out will have centimeter labels and the second ruler will NOT have labels?
- (n) A box contains 7 large red marbles, 5 large yellow marbles, 3 small red marbles, and 5 small yellow marbles. If a marble is drawn at random, what is the probability that it is yellow, given that it is one of the large marbles?

## Short Answer Qs

- (1) Lesley-Anne estimates that she has a 75% chance of passing physics and an 80% chance of passing English. Assuming that {passing English} and {passing Physics} are independent events. What is the probability that Lesley-Anne will pass only one of these two subjects?
- (2) If a satellite launch has a 97% chance of success, what is the probability of three consecutive successful launches?
- (3) Carrie is a kicker on her rugby team. She estimates that her chances of scoring on a penalty kick during a game are 75% when there is no wind, but only 60% on a windy day. If the weather forecast gives a 55% probability of windy weather today, what is the probability of Carrie scoring on a penalty kick in a match this afternoon?
- (4) A bag contains three white marbles, five green marbles, and two red marbles. What is the probability of randomly picking both red marbles in the first two tries? Assume that the first marble picked is not put back into the bag.
- (5) If the probability of the Rangers defeating the Eagles in a hockey game is  $\frac{3}{7}$ , what is the probability that the Rangers will win two consecutive games against the Eagles?
- (6) Statsville has two computer-controlled traffic lights on the road between the main street and the highway. The probability of getting a red light at the first traffic light is 0.45, and the probability of getting a red light at the second one is 0.20 if you had been stopped by a red light at the first one. What is the probability of being stopped by red lights at both intersections?

## Problems

- (1) A survey at a school asked students if they were ill with a cold or the flu during the last month. The results were as follows. None of the students had both a cold and the flu.

Use these results to estimate the probability that:

	Cold	Flu	Healthy
Females	32	18	47
Males	25	19	38

- a) a randomly selected student had a cold in the last month
  - b) a randomly selected female student was healthy last month
  - c) a randomly selected student who had the flu last month is male
  - d) a randomly selected male student had either a cold or the flu last month
- (2) To get out of jail free in the board game MONOPOLY®, you have to roll doubles with a pair of standard dice.
    - a) Determine the **probability** of getting out of jail on your first or second roll.
  - (3) At an athletic event, athletes are tested for steroids using two different tests. The first test has a 93.0% probability of giving accurate results, while the second test is accurate 87.0% of the time. For a sample that does contain steroids, what is the probability that a) neither test shows that steroids are present? b) both tests show that steroids are present? c) at least one of the tests detects the steroids?
  - (4) A test for the presence of E. coli in water detects the bacteria 97% of the time when the bacteria is present, but also gives a false positive 2% of the time, wrongly indicating the presence of E. coli in uninfected water. If 10%

of the water samples tested contain E. coli, what is the probability that a test result indicating the presence of the bacteria is accurate?

- (5) According to Mr. Laposky, the probability that a CAC Grade 10 student registers to take IB Chemistry is 0.4.

Likewise, Mr L also knows that  $P(B|C') = 0.5$  and  $P(B|C) = 0.6$

- a) Construct a tree diagram, showing the outcomes and their associated probabilities. The two events are:  
EVENT #1 → taking Chemistry and EVENT #2 → taking Biology
- b) Determine the probability of a CAC Grade 10 student takes BOTH Chemistry AND Biology.
- c) Find the value of  $P(B|C')$ . Interpret the meaning of the resultant answer.
- d) How probable is it that a CAC Grade 10 student takes Biology?
- e) Given your answer in question (d), determine the value of  $P(B \cap C) = P(B) \times P(C)$ . Compare this value to your answer to Q(b). Offer an explanation as to why.
- f) Determine the value of  $P(B|C) + P(B|C')$ . Interpret the meaning of the resultant answer.
- g) Determine the value of  $P(B \cap C') + P(B \cap C)$ . Interpret the meaning of the resultant answer.
- h) Now, investigate the following calculations:
  - (a)  $P(C) \times P(B|C)$ . Interpret.
  - (b)  $P(C \text{ and } B)$ . Interpret.
  - (c)  $P(C \cap B)$ . Interpret.
  - (d)  $P(C) \times P(B|C) + P(C') \times P(B|C')$ . Interpret.
- i) Is  $P(C|B) = P(B|C)$ ? Show the mathematics supporting your answer.
- j) If there one hundred students in Grade 10 at CAC, construct a Venn diagram showing these Grade 10 students and their IB course selections (with respect to Biology and Chemistry)