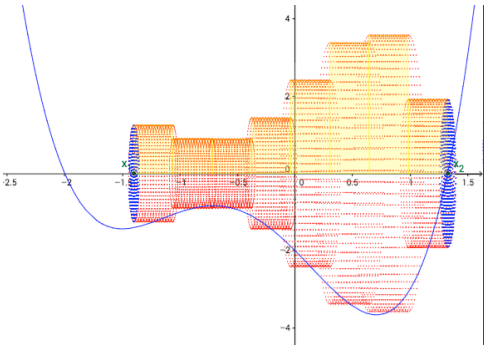
A. Opening Exercise

1. Determine the area of a circle if the radius is 6 cm.
2. Determine the area of a circle if its radius is defined by y = 2x at the point where x = 3.
3. Draw the function f(x) = 2x on the interval [0,3]. Estimate the area under f(x) on [0,3] using 3 rectangles (right side). Draw a diagram
4. Explain what happens when each of the 3 rectangles is completely rotated around the x-axis. Draw a diagram.
5. What shape does each rectangle now have? Determine the volume of each one.
6. Explain what the idea of "volume of revolution" means. What is our estimated volume of revolution?

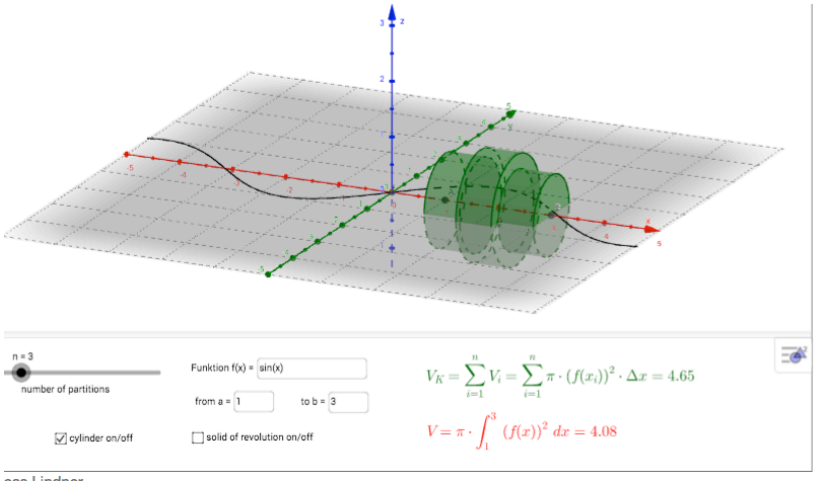
B. Geogebra Animations of VoR

To visualize the concept, let's visit the following geogebra animations:

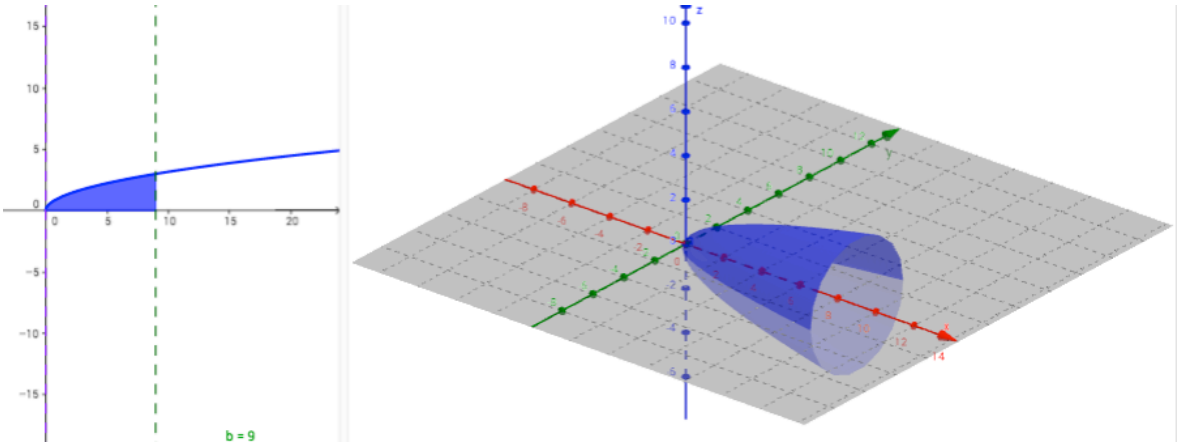
https://www.geogebra.org/m/J6zfMMCV



https://www.geogebra.org/m/uew3jQRg



https://www.geogebra.org/m/mzWq2Cet



C. Volumes of Revolution : Exampes

GENERAL FORMULA ➔

1. The function $f(x) = 2x,\ 1 \le x \le 5$ is rotated about the x-axis to form a solid of revolution. Find the volume of this solid.

2. Find the volume of the solid formed by revolving the region enclosed by the curve with equation $f(x) = \sqrt{x-5},\ 5 \le x \le 9$ about the x-axis.

3. Graph the function $f(x) = 9 - (x-2)^2$. Determine
   i. The area of the region enclosed by the function and the *x*-axis.
   ii. The volume of the solid formed when this area is rotated around the *x*-axis.

4. Find the volume of the solid formed by revolving the region enclosed by the curve with equation $f(x) = \sqrt{25 - x^2}$ about the x-axis. (CA)

5. Find the volume of the solid formed by revolving the region enclosed by the curve with equation $f(x) = \sqrt{\cos(x)\sin(x)},\ x \in \left[0, \frac{\pi}{2}\right]$ about the x-axis. (CA)

D. Further Practice

1. Find the volume of the solid of revolution generated when the area described is rotated about the x-axis.

   (a) The area between the curve $y = x$ and the ordinates $x = 0$ and $x = 4$.

   (b) The area between the curve $y = x^{3/2}$ and the ordinates $x = 1$ and $x = 3$.

   (c) The area between the curve $x^2 + y^2 = 16$ and the ordinates $x = -1$ and $x = 1$.

   (d) The area between the curve $x^2 - y^2 = 9$ and the ordinates $x = -4$ and $x = -3$.

   (e) The area between the curve $y = (2 + x)^2$ and the ordinates $x = 0$ and $x = 1$.

2. The area between the curve $y = 1/x$, the y-axis and the lines $y = 1$ and $y = 2$ is rotated about the y-axis. Find the volume of the solid of revolution formed.

3. The area between the curve $y = x^2$, the y-axis and the lines $y = 0$ and $y = 2$ is rotated about the y-axis. Find the volume of the solid of revolution formed.

4. The area cut off by the x-axis and the curve $y = x^2 - 3x$ is rotated about the x-axis. Find the volume of the solid of revolution formed.

5. Sketch the curve $y^2 = x(x - 4)^2$ and find the volume of the solid of revolution formed when the closed loop of the curve is rotated about the x-axis.

6. A conical funnel is formed by rotating the curve $y = \frac{1}{3}x$ about the y-axis. The radius of the rim of the funnel is to 6 cm. Find the depth of the funnel and its volume.

Answers

1. (a) $21\frac{1}{3}\pi$ (b) 20π (c) $31\frac{1}{3}\pi$ (d) $3\frac{1}{3}\pi$ (e) $\frac{211}{5}\pi$
2. $\frac{1}{2}\pi$
3. 2π
4. $\frac{81}{10}\pi$
5. $21\frac{1}{3}\pi$
6. 24π