A. Properties of Integrals

1. Using the function $f(x) = 4 - x^2$, evaluate the following definite integrals using your TI-84

   i. $\int_0^2 (4 - x^2)\,dx$ and $\int_2^0 (4 - x^2)\,dx$

   ii. $\int_0^3 (4 - x^2)\,dx$ and $\int_3^0 (4 - x^2)\,dx$

   iii. $\int_0^\pi \sin(x)\,dx$ and $\int_\pi^0 \sin(x)\,dx$

   iv. And the point being made is …… ?

2. So, looking for patterns ➔ here's one ➔ Evaluate the following 4 definite integrals:

   i. First $\int_0^5 (x^2 - 4)\,dx$

   ii. and then $\int_0^2 (x^2 - 4)\,dx + \int_2^5 (x^2 - 4)\,dx$ (why did I select x = 2 to "split up" the integral?

   iii. And the point being made is?

   iv. then $\int_0^2 |x^2 - 4|\,dx + \int_2^5 |x^2 - 4|\,dx$

   v. and then $\int_0^5 |x^2 - 4|\,dx$

   vi. and the point being made is …… ?

B. Area Between Curves

1. Evaluate the following definite integral ➔ $\int_0^2 x^2 dx$. Verify and draw using a GDC

2. Evaluate the following definite integral ➔ $\int_0^2 2x dx$. Verify and draw using a GDC

3. HENCE, determine the area between the curves $f(x) = 2x$ and $g(x) = x^2$ between $x = 0$ and $x = 2$.

4. Evaluate the following definite integral ➔ $\int_0^2 (2x - x^2) dx$ and then $\int_0^2 2x dx - \int_0^2 x^2 dx$.

5. What is the significance of using $x = 0$ and $x = 2$ in this problem?

6. What point(s) is/are being made by this example?

7. Find the area between the curves $f(x) = 3x$ and $g(x) = 4 - x^2$. Set the question up with your TI-84 to help visualize the problem (but even more ideal would be doing without the GDC) and then answer the question ALGEBRAICALLY. Either way, use the Ti-84 to verify your answer.

8. Find the area between the curves $f(x) = x + 2$, $g(x) = x^2 + x - 2$ and the lines $x = -1$ and $x = 1$. Set the question up with your TI-84 to help visualize the problem (but even more ideal would be doing without the GDC) and then answer the question ALGEBRAICALLY. Either way, use the Ti-84 to verify your answer.

9. **FURTHER PRACTICE**

   i. Find the area bounded by $f(x) = -x^2 + 1, g(x) = 2x + 4, \mathbf{x = -1, and}\ x = 2$

   ii. Find the area between the curves $y = \sqrt{x}$ **and** $y = x^3$

   iii. Find the region enclosed by $h(x) = x^2 - 2x$ **and** $k(x) = x$ **on** $[0,4]$

   iv. Find the region enclosed by $y = \sin x$ **and** $y = \pi x - x^2$

C. Integrals: Method of Substitution

Looking for Patterns

1. Take the derivative of the following functions:
   i. $k(x) = \ln(3x - 5)$
   ii. $f(x) = e^{x^2}$
   iii. $g(x) = \sin(3x^2)$
   iv. $h(x) = \sqrt{x^4 + 5}$

2. Given your work in PART 1, now determine the integrals of the following:
   i. $\int \frac{3}{3x-5} dx$
   ii. $\int 2xe^{x^2} dx$
   iii. $\int 6x\cos(3x^2) dx$
   iv. $\int \frac{2x^3}{\sqrt{x^4+5}} dx$

   v. What pattern do you see being repeated every time?

   vi. So, how can we "undo" the chain rule (How do we work with composite functions?)

Additional practice

http://www.teaching.martahidegkuti.com/shared/lnotes/6_calculus/integral/substitution/substitutionb.pdf