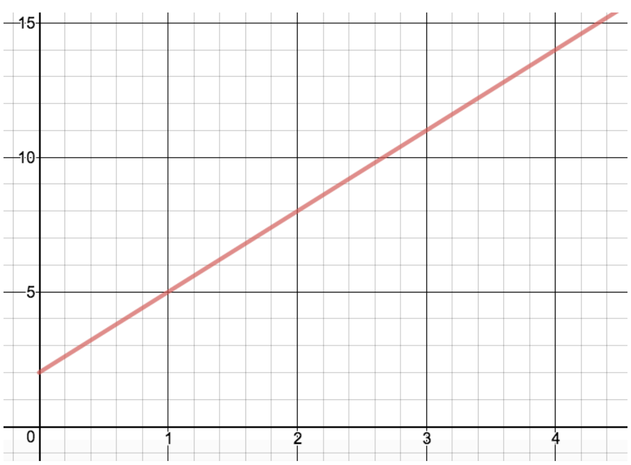
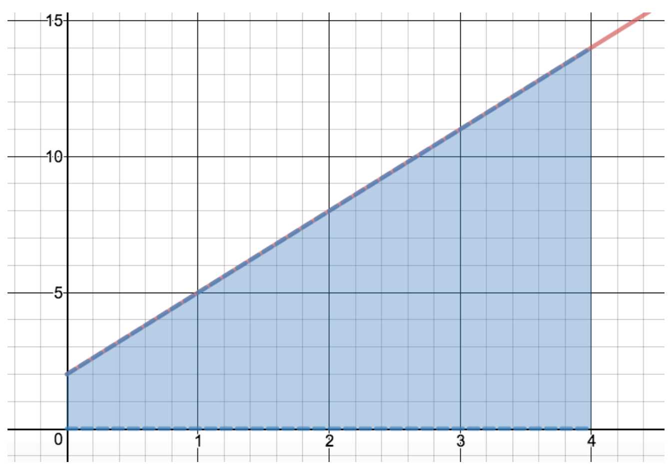
A. Lesson Context

BIG PICTURE of this UNIT:	• How do we measure "change" in a function or function model? • Why do we measure "change" in a function? • How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level?		
CONTEXT of this LESSON:	Where we've been We have connected two ideas through our motion work ➔ antiderivatives and area under the curve	Where we are Can we now carry forward the area under the curve idea to nonlinear functions and once again use an antiderivative?	Where we are heading We will explore the concept of integration

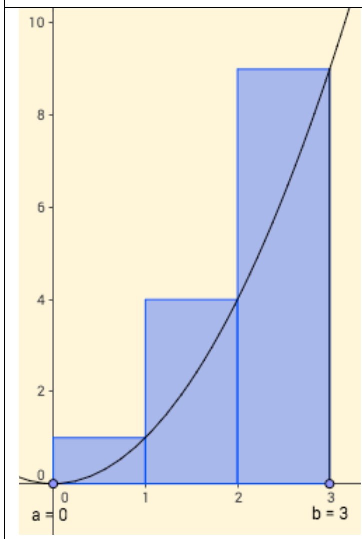
B. Recap:

We have connected two fundamental ideas in our last lesson ➔ idea #1: that of area under a curve (in our simplest cases wherein we looked at a velocity-time graph) to find a total distance travelled (or displacement) AND then idea #2: where we can use an antiderivative (i.e the position function) to find the same answer of distance traveled (or displacement).

An object travels with a velocity defined by the function $v(t) = 3t + 2$ for 4 seconds. Its starting position was $s(0) = 2$. How far did it travel?	
Velocity-Time graph 	Area under the curve of V-T graph  Total distance travelled is (2x4) + 0.5(4)(12) = 32 m
Using the antiderivatives, the position function is $s(t) = 1.5t^2 + 2t + 2$. So we can also use this position function to determine $s(4) – s(0) = (1.5x4^2 + 2x4 + 2) – 2 = 24 + 8 + 2 – 2 = 32$ and come up with the same 32 m of distance traveled.	

C. <u>But what if …….</u>

But what happens when the velocity function is NOT linear, but rather a curve? How do we now find an area under the curve?

<table>
<tr><td colspan="3">So, let's estimate the area under the curve of f(x) = x^2, between x = 0 and x = 3 ➔ how? ➔ let's make rectangles</td></tr>
<tr>
<td>
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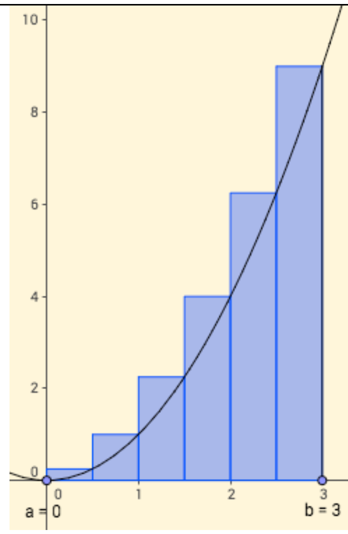
<td>
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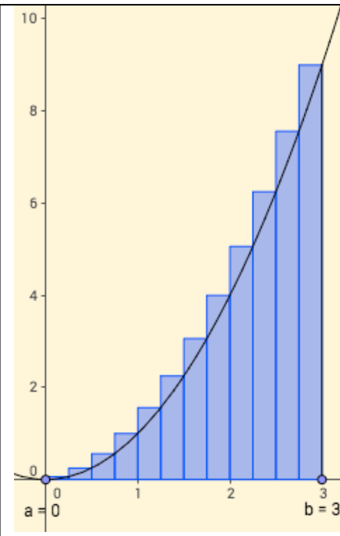
<td>
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<tr>
<td>$A_T = A_1 + A_2 + A_3$

AT = $h_1w + h_2w + h_3w$

$A_T = f(1)\cdot 1 + f(2)\cdot 1 + f(3)\cdot 1$</td>
<td>$A_T = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$

AT = $h_1w + h_2w + h_3w + h_4w + h_5w + h_6w$

$A_T = f(0.5)\cdot \frac{1}{2} + f(1)\cdot \frac{1}{2} + ... + f(2.5)\cdot \frac{1}{2} + f(3)\cdot \frac{1}{2}$</td>
<td>$A_T = A_1 + A_2 + A_3 + ...\ A_7 + A_8 + A_9$

AT = $h_1w + h_2w + \ ....\ + h_8w + h_9w$

$A_T = f\left(\frac{1}{3}\right)\cdot \frac{1}{3} + f\left(\frac{2}{3}\right)\cdot \frac{1}{3} + ... + f\left(\frac{8}{3}\right)\cdot \frac{1}{3} + f(3)\cdot \frac{1}{3}$</td>
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</table>

So what are we seeing? A summation of a the areas of rectangles (and the dimensions of each rectangle are determined by the function "height" multiplied by a width)

$$A_T = A_1 + A_2 + A_3 + ... = \sum_{i=1}^{n} A_i$$

$$A_T = f(x_1)\cdot \Delta x + f(x_2)\cdot \Delta x + f(x_3)\cdot \Delta x + ... = \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

But how many rectangles do we need? ➔ how about an infinite number!!! ➔ hence that limit idea again …….

$$A_T = f(x_1)\cdot \Delta x + f(x_2)\cdot \Delta x + f(x_3)\cdot \Delta x + ... = \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

$$A_T = \lim_{n \to} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$$

So how about a new symbol? $A_T = \lim_{n\to} \sum_{i=1}^{n} f(x_i)\cdot \Delta x$ will now be represented/replaced by $A_T = \int_a^b f(x)dx$, where *a* and *b* are the two x-value "boundaries" along the x-axis that form the area we are after.

What calculus approach do we use to find these areas? ➔ Antiderivatives!!

So what does the "integral" symbol ask us to "perform"? ➔ determine an antiderivative!!

D. Indefinite Integrals as Antiderivatives

1. Notice that these two columns are really asking the SAME question, asking the same thing of you

(a) Find the antiderivative of x^4	(a) Find the indefinite integral $\int x^4 dx$
(b) Find the antiderivative of $x^{\frac{2}{5}}$	(b) Find the indefinite integral $\int x^{\frac{2}{5}} dx$
(c) Find the antiderivative of $\frac{1}{\sqrt[5]{x}}$	(c) Find the indefinite integral $\int \left(\frac{1}{\sqrt[5]{x}}\right) dx$

2. Find the following indefinite integrals:

   i. $\int x^3 dx$ and then $\int 2x^3 dx$ and then $\int -4x^3 dx$ and then $\int \frac{1}{\sqrt[3]{2}} x^3 dx$ ➔ point being?

   ii. $\int x^3 dx$ and then $\int 5x^{-2} dx$ and then $\int 4 dx$ and then finally $\int (x^3 + 5x^{-2} + 4) dx$ ➔ point being?

   iii. $\int x^3 dx$ and then $\int (x+2)^3 dx$ and then $\int (x-4)^3 dx$ and then $\int (x-\pi)^3 dx$ ➔ point being?

   iv. $\int x^3 dx$ and then $\int 2x^3 dx$ and then $\int -4x^3 dx$ and then $\int \frac{1}{\sqrt[3]{2}} x^3 dx$ ➔ point being?

   v. $\int \frac{1}{x} dx$ and then $\int \frac{2}{x} dx$ and then $\int e^x dx$ and then $\int -4e^x dx$ ➔ point being?

   vi. $\int \frac{1}{2x} dx$ and then $\int \frac{1}{x+2} dx$ and then $\int \frac{1}{3x+2} dx$ and then $\int \frac{4}{4-3x} dx$ ➔ point being?

   vii. $\int \sin(x) dx$ and then $\int 2\cos(x) dx$ and then $-\int \sin(x) dx$ and then $\int \cos(x) dx$ ➔ point being?

   viii. $\int \sin(x+2) dx$ and then $\int \sin(x-2) dx$ and then $\int \sin(3x) dx$ and then $\int \sin(3x-5) dx$ ➔ point being?

http://www.personal.kent.edu/~bosikiew/Math11012/Section5-1.pdf

E. **Definite Integrals as Area under the curve using antiderivatives: EXAMPLES**

1. Evaluate the following definite integral → $\int_{1}^{5}(x-1)dx$. Verify using a GDC
2. Evaluate the following definite integral → $\int_{-5}^{1}(x-1)dx$. Verify using a GDC
3. Evaluate the following definite integral → $\int_{-5}^{5}(x-1)dx$. Verify using a GDC

What point(s) is/are being made by these three examples?

4. Evaluate the following definite integral → $\int_{1}^{3}\left(x^2+2x\right)dx$. Verify using a GDC
5. Evaluate the following definite integral → $\int_{1}^{4}\left(\frac{1}{x}\right)dx$. Verify using a GDC
6. Evaluate the following definite integral → $\int_{1}^{3}4x^2(x+1)\,dx$. Verify using a GDC
7. Evaluate the following definite integral → $\int_{0}^{\pi/2}\sin(x)\,dx$. Verify using a GDC
8. Evaluate the following definite integral → $\int_{0}^{\frac{3\pi}{2}}\cos(x-\pi)\,dx$. Verify using a GDC

http://hahnmath.weebly.com/uploads/5/7/6/3/57632417/worksheet_-_definite_integrals.pdf