

**A. Lesson Context**

BIG PICTURE of this UNIT:	<ul style="list-style-type: none"> <li>• How do we measure “change” in a function or function model?</li> <li>• Why do we measure “change” in a function?</li> <li>• How do we analytically analyze a function or function model – beyond a simple preCalculus &amp; visual/graphic level?</li> </ul>		
CONTEXT of this LESSON:	<p>Where we’ve been</p> <p>We understand how to differentiate and work with polynomial , sinusoidal, exponential &amp; log functions &amp; we understanding new differentiation techniques</p>	<p>Where we are</p> <p>Can we now consolidate our skills in differentiating and working with variety of functions?</p>	<p>Where we are heading</p> <p>Working with more complicated functions that are variations of polynomials, sinusoidal and exp/log</p>

**B. Review – Differentiation Techniques: Summary**

Rule	“formula”
Chain Rule	
Product Rule	
Quotient Rule	
Derivative of trig fcns:	
Derivative of Exp & log fcns:	
Derivative of power fcns:	

**C. Applying Calculus: Working with Rates of Change & Tangents & Normals**

- (CI) Given the function  $f(x) = \ln(2x - 1)$ , determine:
  - The equation of the derivative.
  - The exact value of the instantaneous rate of change at  $x = 2$ .
  - At what point(s) does the function have an instantaneous rate of change of  $\frac{1}{3}$ ?
- (CI) Given the function  $f(x) = (3x + 1)\ln(x)$ , determine:
  - The equation of the derivative
  - The exact value of the instantaneous rate of change at  $x = e$ .
- (CI) Determine the equation of the line tangent to  $y = (6x + 3)^{\frac{5}{3}}$  at  $x = 4$ .
- (CI) Find  $\frac{dy}{dx}$  for  $y = \frac{x^2 - 1}{2x^2 + 1}$ . Determine the values of  $x$  for which  $\frac{dy}{dx} > 0$ .
- (CI) Determine the interval(s) in which the curve of  $g(x) = \sin^2(x)$  is concave down.
- (CI) Find the equation of the line normal to the curve of  $y = 4xe^{x^2 - 4}$  at the point  $(2, 8)$ .
- (CI) Determine the equation of the line tangent to  $y = x^3 \ln(x)$  at  $x = 1$ .
- (CI) Determine the equation of the tangent to  $y = \cos(\pi - 2x)$  at the point where  $x = \frac{\pi}{4}$ .
- (CI) Find the point(s) where the tangent line to the curve of  $f(x) = e^{2x - 3x^2}$  is horizontal.
- (CI) Find the point(s) where the tangent line to the curve of  $f(x) = \frac{x^2 - 2x + 4}{x^2 + 4}$  is horizontal.

11. (CI) Find the minimal value of  $g(x) = \frac{e^x}{x}$ ;  $x > 0$ .

12. (CI) Determine the equation of the tangent to  $y = (2x^3 - 4x + 2)(x^2 - 3x + 1)$  at the point  $(2, -10)$ .

13. (CI) For the curve defined by  $g(x) = (\sin(x) + \cos(x))^2$  on the domain of  $[0, 2\pi]$ , determine:

- i. the x- and y-intercept(s);
- ii. Show that  $g'(x) = 2\cos(2x)$
- iii. the first three stationary points;
- iv. the “nature” of these stationary points (max/min/neither);
- v. hence, sketch the function  $g(x) = (\sin(x) + \cos(x))^2$ .

14. (CI) Given the function  $y = \sqrt{6x - 5}$ .

- i. State the domain for the function  $y = \sqrt{6x - 5}$ .
- ii. Find  $\frac{dy}{dx}$  for  $y = \sqrt{6x - 5}$ .
- iii. HENCE, explain why  $\frac{dy}{dx} > 0$  for all values of  $x$  of the domain.
- iv. What does this mean about the function?

15. (CI) For the curve defined by  $g(x) = e^{-x} \cos(x)$  on the domain of  $\left[0, \frac{5\pi}{2}\right]$ , determine:

- i. the x- and y-intercept(s);
- ii. the first two stationary points;
- iii. the “nature” of these stationary points (max/min/neither);
- iv. hence, sketch the function  $g(x) = e^{-x} \cos(x)$ .

16. (CI) Given the function  $g(x) = \cos(x) + \frac{1}{2}\cos(2x)$ ,  $x \in [0, 2\pi]$ , sketch the graph, identifying all important features including maximum(s), minimum(s) and inflection points.

17. (CI) For the curve of  $g(x) = e^x \sin(x)$ ,

- i. Find the equations of  $g'(x)$  and  $g''(x)$ .
- ii. Find the values of  $x$  for which  $g'(x) = 0$  and  $g''(x) = 0$ .
- iii. Given your work in Qi. and Qii., determine the intervals of increase and decrease and classify the extrema.
- iv. Sketch the function, given your work in Qiii.

18. (CI) Given the function  $g(x) = xe^x - e^x$ ,

- i. Evaluate the exact values of: (a)  $g(1)$  (b)  $g(0)$
- ii. Show that  $\frac{d}{dx} g(x) = e^x + g(x)$ .
- iii. Determine the intervals of increase/decrease of  $y = g(x)$ .
- iv. Show that  $y = g(x)$  has an inflection point at  $x = -1$
- v. Determine the interval in which  $y = g(x)$  is concave up.
- vi. Determine the equation of the tangent to the curve at  $x = 1$ .

19. (CI) Given the function  $y = \frac{x}{3-2x}$ ;

- i. Determine the equation of the asymptotes of this rational function.
- ii. Hence or otherwise, evaluate  $\lim_{x \rightarrow -\infty} \frac{x}{3-2x}$
- iii. Determine the x- and y-intercepts.
- iv. Find the equation of the line that is normal to the curve  $y = \frac{x}{3-2x}$  at the point where  $x = 1$ .