SL2 - Lesson 13: Differentiation of Sinusoidal, Exponential and Logarithmic Functions | Topic 6 – Calculus

A. Lesson Context

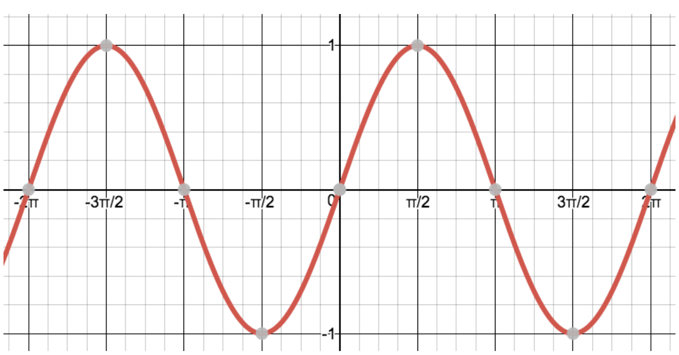
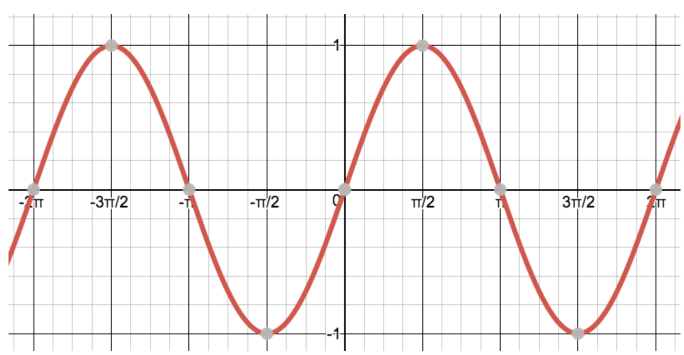
BIG PICTURE of this UNIT:	• How do we measure "change" in a function or function model? • How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level? • ?		
CONTEXT of this LESSON:	Where we've been We understand how to differentiate and work with polynomial functions	Where we are How do we differentiate and work with sinusoidal and exponential and logarithmic functions?	Where we are heading Working with more complicated functions that are variations of polynomials, sinusoidal and exp/log

B. Lesson Objectives

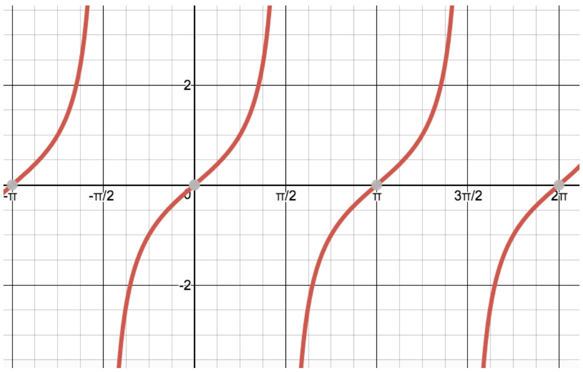
1. Use derivative sketching basics in order to graph the derivatives of 5 "new" functions
2. Review the key points/features of the parent functions
3. State the derivatives of the "new" functions.
4. Use these derivatives in order to practice calculus skills with these "new" functions (tangents/normals & curve sketching)

C. Problem 1 – Skill Development – Derivatives of sin(x), cos(x), tan(x), e^x and ln(x)

Here are the graphs of the 5 "new" functions we wish to differentiate. Use your knowledge of derivatives and thereby sketch graphs of the derivatives of these 5 functions.

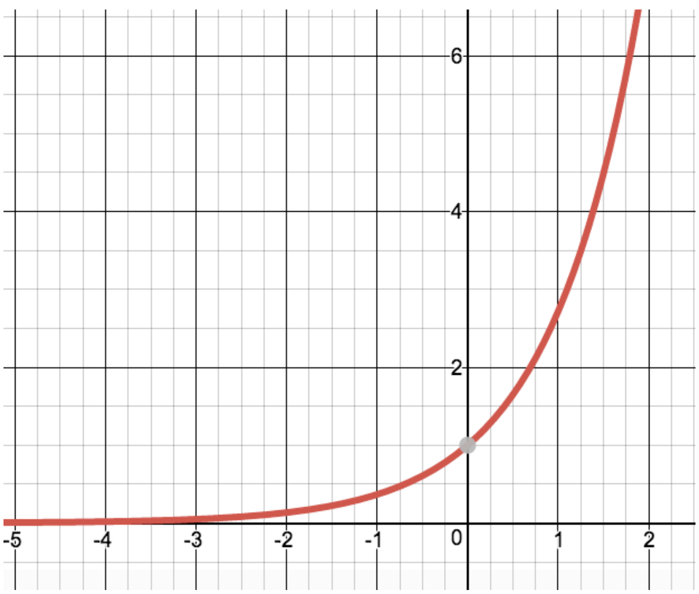
f(x) = sin(x)	f(x) = cos(x)
	
Key Points: (exact & approx.)	Key Points: (exact & approx.)

g(x) = tan(x)



Key points for g(x) = tan(x)   (exact & approx.)

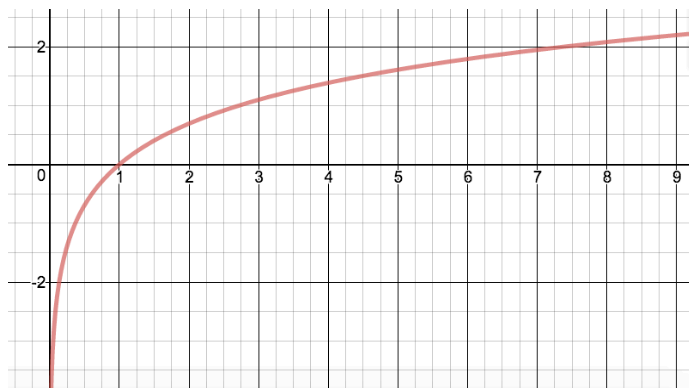
h(x) = e^x



Key Points: (exact & approx.)

Algebra "trick" ➔ $e^{\ln(a)} = ?$

k(x) = ln(x)



Key points: (exact & approx.)

Algebra "trick" ➔ $\ln(e^a) = \ln(e)^a = ?$

Now, let's use wolframalpha.com and your TI-84 GDC to find/confirm the equation of the derivatives of these 5 functions:

function	f(x) = sin(x)	f(x) = cos(x)	f(x) = tan(x)	f(x) = e^x	f(x) = ln(x)
derivative					

D. Problem 2 – Applying Calculus: Working with Rates of Change & Tangents & Normals

1. Given the function $f(x) = 5\sqrt{x} - \frac{1}{5}e^x$, determine:
   i. The equation of the derivative
   ii. The exact value of the instantaneous rate of change at x = 4.

2. Find the equation of the derivative of the following functions:
   i. $h(x) = 3\sin(x) - 2\cos(x)$
   ii. $h(x) = \tan(x) - \ln\left(x^2\right)$

3. Evaluate $e^{\ln(x)} + e^{\ln\left(x^2\right)} - 2e^{\ln\left(\frac{1}{x^3}\right)}$.

4. (**CI**) Given the function $g(x) = x - \ln(x)$,
   i. Evaluate the exact values of: (a) $g(1)$ (b) $g(2)$
   ii. Show that $\frac{d}{dx}g(x) = \frac{x-1}{x}$.
   iii. Hence, detemine the intervals of increase/decrease of $y = g(x)$.
   iv. Show that $y = g(x)$ is concave up on its entire domain.
   v. Determine the equation of the tangent to the curve at *x* = 2.

5. (CI) Determine the equation of the line tangent to $y = \sin(x) - \cos(x)$ at $x = \frac{\pi}{2}$.

6. (CI) Determine the equation of the line that is tangent to $y = 4e^x - 7$ at $x = \ln(3)$.

7. (CI) Determine the equation of the line that is orthogonal to $f(x) = 2\tan(x)$ at $x = \frac{\pi}{4}$.

8. (CI) For the curve defined by $g(x) = \sin(x) + \cos(x)$, determine:

   i. the x- and y-intercept(s);
   ii. the intervals of increase and decrease;
   iii. the critical points;
   iv. the intervals of concavity;
   v. hence, sketch the function $g(x) = \sin(x) + \cos(x)$.

9. (CI) For the curve defined by $g(x) = \ln\left(x^2\right) - \frac{1}{x}$, determine:

   i. the x- and y-intercept(s) and the asymptote(s);
   ii. the intervals of increase and decrease;
   iii. the critical points;
   iv. the intervals of concavity;
   v. hence, sketch the function $g(x) = \ln\left(x^2\right) - \frac{1}{x}$.

10. Determine the equation of the line normal to the curve of $g(x) = \sqrt[3]{x} + \ln(x) + 2$ at

   i. (**CI**) the point where *x* = 1.
   ii. (**CI**) Show that $g'(e) = \frac{\sqrt[3]{e} + 3}{3e}$.
   iii. (**CA**) at the point wher *x* = *e*.