Lesson 6 – Derivatives As Functions

IB Math SL2 - Santowski

Math SL2- Santowski

8/27/16

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Lesson Objectives

- 1. Introduce the concept of the derivative
- 2. Calculate the derivative functions of simple polynomial functions from first principles
- 3. Calculate the derivative of simple polynomial functions using the TI-84
- 4. Calculate derivatives and apply to real world scenarios

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(A) The Derivative at a Point

- We will introduce a new term to describe this process of calculating the tangent slope (or calculating the instantaneous rate of change)
- We will now call this a DERIVATIVE at a point (for reasons that will be explained at the END of the lesson)

$$m_{\text{tangent}} = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

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(A) The Derivative as a Function

- Our second change now will be to alter our point of view and let the value of *a* or *x* (the point at which we are finding the derivative value) vary (in other words, it will be a variable)
- Consequently, we develop a new function - which we will now call the derived function (AKA the derivative)
- We will do this as an investigation using two different methods: a graphic/numeric approach and a more algebraic approach

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(A) The Derivative as a Function

- Choose your own values of a, b, c for a quadratic equation.
- Make sure your quadratic eqn is different than others in class.
- Ex: f(x) = x^2 - 4x - 8 for the interval [-3,8]

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(A) The Derivative as a Function

- We will work with foundational concepts - the tangent concept – and draw tangents to given functions at various points, tabulate results, create scatter-plots and do a regression analysis to determine the equation of the curve of best fit.
- Ex: f(x) = x^2 - 4x - 8 for the interval [-3,8]

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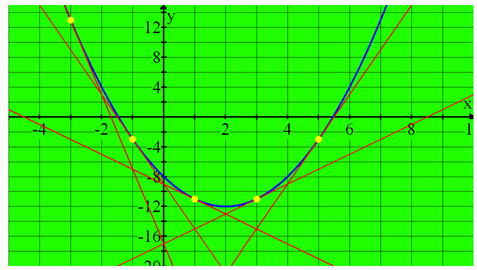
(A) The Derivative as a Function

- Example: $y = x^2 - 4x - 8$. for the interval [-3,8]
- 1. Draw graph.
- 2. Find the tangent slope at x = -3 using the TI-89
- 3. Repeat for x = -2,-1,….,7,8 and tabulate

X	-3	-2	-1	0	1	2	3	4	5	6	7	8
Slope	-10	-8	-6	-4	-2	0	2	4	6	8	10	12

- 4. Tabulate data and create scatter-plot
- 5. Find best regression equation

(A) The Derivative as a Function



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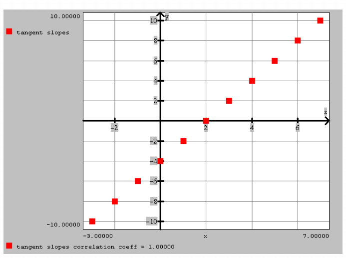
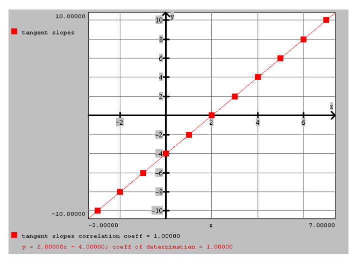


(A) The Derivative as a Function



(A) The Derivative as a Function

- Our function equation was f(x) = $x^2 - 4x - 8$
- Equation generated is g(x) = 2x - 4
- The interpretation of this "derived equation" is that this "formula" (or equation) will give you the slope of the tangent (or instantaneous rate of change) at every single point *x*.
- The equation g(x) = 2x - 4 is called the derived function, or the derivative function of f(x) = $x^2 - 4x - 8$

(B) The Derivative as a Function - Algebraic

- Given f(x) = $x^2 - 4x - 8$, we will find the derivative at x = a using our "derivative formula" of

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- Our one change will be to keep the variable *x* in the "derivative formula", since we do not wish to substitute in a specific value like *a*

(B) The Derivative as a Function - Algebraic

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$$f'(a)=\lim_{h\to 0}\frac{\left((a+h)^2-4(a+h)-8\right)-\left(a^2-4a-8\right)}{h}$$

$$f'(a)=\lim_{h\to 0}\frac{\left(a^2+2ah+h^2-4a-4h-8\right)-\left(a^2-4a-8\right)}{h}$$

$$f'(a)=\lim_{h\to 0}\frac{2ah+h^2-4h}{h}$$

$$f'(a)=\lim_{h\to 0}(2a+h-4)$$

$$f'(a)=2a-4$$

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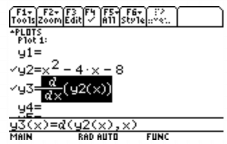
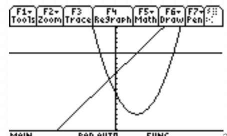
(B) The Derivative as a Function – TI-84

0 

0 

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Calculus Applet – Concept Visualizations

0 Animation #1 → https://www.geogebra.org/m/hezRxyqz

0 Animation #2 → https://www.geogebra.org/m/MeMdCUEm

0 Amination #3 → http://mathinsight.org/applet/derivative_function

0 Animation #4 → https://www.maa.org/sites/default/files/images/upload_library/4/vol4/kaskosz/derapp.html

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(C) In-Class Examples – Matching Graphs

0 Go to the following sites and match the graphs of functions with the graphs of their derivatives

0 https://www.univie.ac.at/moe/tests/diff1/ablerkennen.html

0 http://www.zweigmedia.com/RealWorld/calctopic1/derivgraphex.html

0 Go to this link for a copy of the matching handout

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(C) In-Class Examples - Algebraic

0 Use the algebraic method to determine the equations of the derivative functions for the following:

$(a)\ f(x)=(2x-7)^2$

$(b)\ g(x)=\dfrac{2}{x-3}$

$(c)\ h(x)=\sqrt{2x-1}$

$(d)\ k(x)=x^3-x$

0 Confirm your derivative equations using the TI-84

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