A. Lesson Context

BIG PICTURE of this UNIT:	• How do we measure "change" in a function or function model? • How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level? • ?		
CONTEXT of this LESSON:	Where we've been In Lesson 2, you were introduced to the concept of average rates of change	Where we are How do we estimate the instantaneous rate of change in a function given a narrowing interval & how do we interpret an instantaneous rate of change in function models	Where we are heading How do we apply the concept of average rates of change to predict and then determine an instantaneous rate of change?

B. Lesson Objectives

a. Review the basic concept of "limits" as it relates to the idea of a "convergence"
b. Understand the idea of "local linearity" and its relationship to average and instantaneous rates of change.
c. Understand and work with the idea of a line tangent to a function.
d. Estimate instantaneous rates of change using a variety of methods

C. Exploring the Idea of Limits

1. Explain the following term ➔ a line that is ***tangent*** to a curve

2. To introduce the concept of ***limits*** ➔ explore the following **sequences**. In each case, you should make the same observation about the term values as the number of terms increases. What do you observe and what predictions can you make?

(a) $2.1,\ 2.01,\ 2.001,\ 2.0001, \ldots\ldots\ldots$

(b) $1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6}, \ldots\ldots\ldots$

(c) $\frac{1}{2},\ \frac{2}{3},\ \frac{3}{4},\ \frac{4}{5},\ \frac{5}{6}, \ldots\ldots\ldots$

(d) $a_n = \frac{n+2}{2n-1}$

3. To introduce the concept of ***limits*** ➔ explore the following **series**. In each case, you should make the same observation about the series sum as the number of terms increases (partial sums). What do you observe and what predictions can you make?

(a) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \ldots\ldots\ldots$

(b) $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\ldots\ldots$

(c) $7 + \frac{21}{4} + \frac{63}{16} + \frac{189}{64} + \ldots\ldots$

(d) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

4. To continue exploring the concept of ***limits*** ➔ explore the following **rational functions** using DESMOS:

   i. Given the function $f(x) = \frac{1}{x-3} + 4$, what happens to the function values (the y values) as:

      1. the input values, x, get larger and larger (as positve values)?
      2. the input values, x, get more and more negative?
      3. Explain what the statement $\lim_{x \to +\infty} f(x)$ might mean, given your work in the previous 2 questions?
      4. the input values, x, get closer and closer to 3 (but are still greater than 3)?
      5. the input values, x, closer and closer to 3 (but are still less than than 3)?
      6. Explain what the statement $\lim_{x \to 3^-} f(x)$ might mean, given your work in the previous 2 questions

D. Exploring the idea of Local "Linearity"

1. Write the equation of a line that goes through the point (2,-3) and has a slope of 2.

2. Use algebra to solve the system defined by $\begin{array}{l} f(x) = (x-1)^2 - 4 \\ g(x) = 2x - 7 \end{array}$. Verify your work using DESMOS.

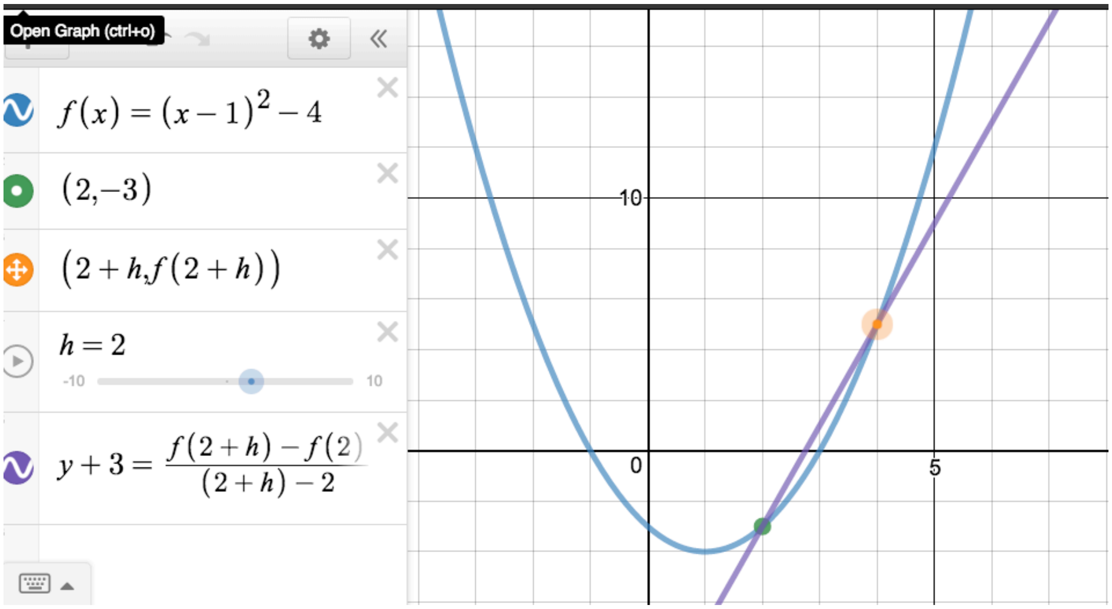
3. Use the TI-84 to graph the parabola f(x) = $(x – 1)^2$ – 4 as well as the point (2,-3). Use TI-84 to start zooming in on the function at the point (2,-3). As you continue to zoom in:

   a. Why does the "curve" start to appear linear?
   b. Using the TI-84, select 2 points on this "linear curve" and calculate the slope.
   c. Using the TI-84, zoom in once more, select 2 points and calculate the new slope.
   d. Using the TI-84, zoom in once more, select 2 points and calculate the new slope.
   e. You should now be able to predict what should happen if we continue to zoom into the "curve" at the point (2,-3). Make your prediction.
   f. Now, with the zoomed in graph, use the TI-84 to graph the line y = 2x – 7.
   g. Describe your observation.

4. Use online resources to help define the term "local linearity".

5. To explore "local linearity" further, graph the following functions on DESMOS: $g(x) = x^3 - x + 2$ and $h(x) = 2|x-2|$. Are there points/locations wherein these functions do or do not demonstrate "local linearity"? Explain your reasoning.

6. Create the following animation using DESMOS (we will work with it next lesson):



E. Further Explorations of Limits

a. Given the function $f(x) = \dfrac{x^2 - 1}{x - 1}$, what happens to the function values (the y values) as:

i. the input values, x, get larger and larger (as positve values)?
ii. the input values, x, get more and more negative?
iii. Explain what the statement $\lim_{x \to +\infty} f(x)$ might mean, given your work in the previous 2 questions?
iv. the input values, x, get closer and closer to 1 (but are still greater than 1)?
v. the input values, x, closer and closer to 1 (but are still less than than 1)?
vi. Explain what the statement $\lim_{x \to 1^-} f(x)$ might mean, given your work in the previous 2 questions

b. Explain the meaning of the mathematical statement $\lim_{x \to c} f(x) = L$ means.

i. HENCE, use DESMOS to explain why/how $\lim_{x \to 3} \dfrac{x^2 - x - 2}{x - 2} = 4$
ii. HENCE, use DESMOS to explain why/how $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x - 2} = 3$
iii. Factor $x^2 - x - 2$ and HENCE explain why $\dfrac{x^2 - x - 2}{x - 2} = x + 1;\ x \neq 2$
iv. HENCE, provide an algebraic explanation for why/how $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x - 2} = 3$

F. HOMEWORK:

Watch the following videos & record notes/examples:

(1) https://www.youtube.com/watch?v=jTvTthjtIrk from MySecretMath Tutor
(2) https://www.youtube.com/watch?v=wpNCdH_Ilmc from Dan Monenineteen

(3) https://www.youtube.com/watch?v=-jJlCX2672M from Bill Hawkins is also very good

Try some questions from Nelson 12 Advanced Functions & Calculus, Chap 3.1, p172- 183, Q1-4 on p178 (http://mrsantowski.tripod.com/2010MathSLY1/Assessments/NelsonS31p172.pdf)