A. Lesson Context

BIG PICTURE of this UNIT:	• How do we measure "change" in a function or function model? • How do we analytically analyze a function or function model – beyond a simple preCalculus & visual/graphic level? • ?		
CONTEXT of this LESSON:	Where we've been In your video HW, you were introduced to the concept of average rates of change	Where we are How do we measure the average rate of change in a function given a specific interval & how do we interpret an average rate of change in function models	Where we are heading How do we apply the concept of average rates of change to predict and then determine an instantaneous rate of change?

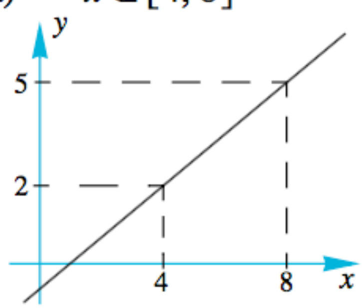
B. Lesson Objectives

a. Review basic function concepts
b. Find the average rate of change of a function, given two points.
c. Apply a knowledge of average rates of change to function models.
d. Introduce the idea of estimating instantaneous rates of change

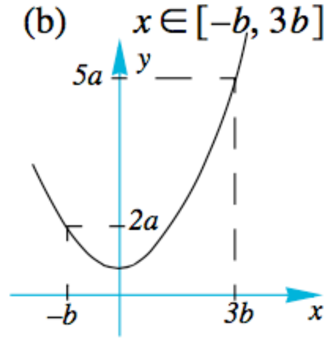
C. Problem 1 – Skill Development

1. For each of the following graphs determine the average rate of change over the specified domain.

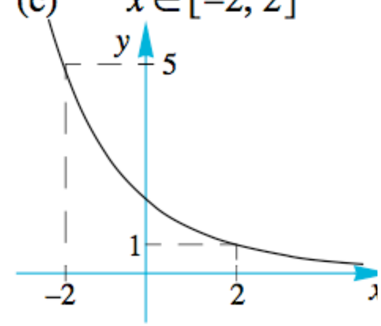
(a) $x \in [4, 8]$



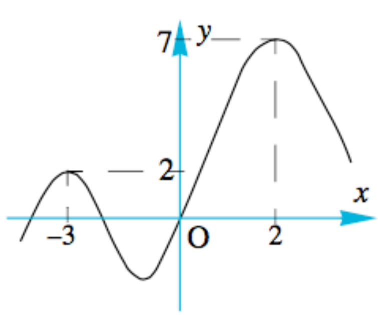
(b) $x \in [-b, 3b]$



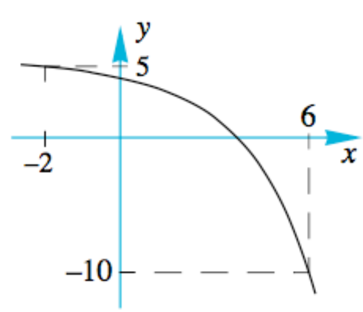
(c) $x \in [-2, 2]$



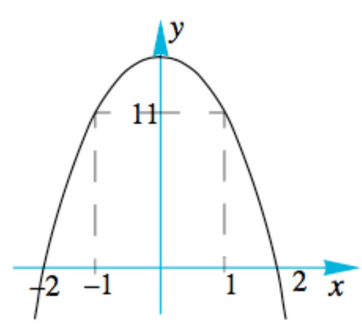
(d) $x \in [-3, 2]$



(e) $x \in [-2, 6]$



(f) $x \in [-1, 1]$



2. For each of the following functions, find the average rate of change over the given domain.

(a) $x \mapsto x^2 + 2x - 1,\ x \in [0, 2]$

(b) $x \mapsto \sqrt{x+1},\ x \in [3, 8]$

(c) $x \mapsto 10 - \dfrac{1}{\sqrt{x}},\ x \in [2, 20]$

(d) $x \mapsto \dfrac{x}{x+1},\ x \in [0.1, 1.1]$

(e) $x \mapsto \dfrac{1}{1+x^2} - 1,\ x \in [0, 100]$

(f) $x \mapsto x\sqrt{400 - x},\ x \in [300, 400]$

(g) $x \mapsto 2^x,\ x \in [0, 5]$

(h) $x \mapsto (x-1)(x+3),\ x \in [-3, 2]$

D. Problem 2 – Consolidating Function Concepts: Parabolas

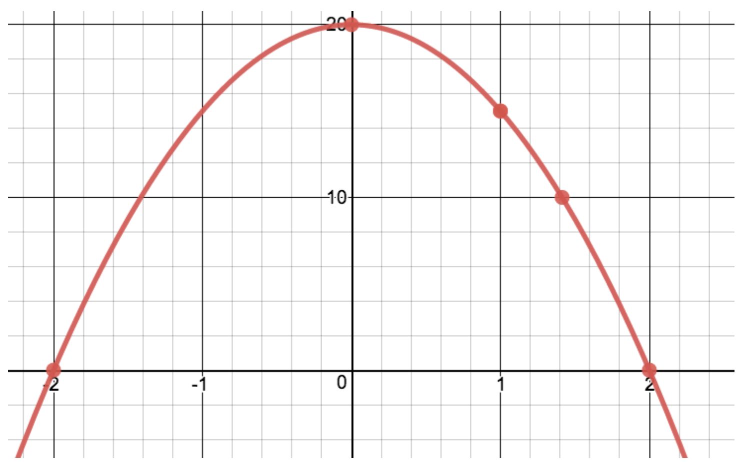
(a) Here is a graph of function, in the form of $f(x) = A + Bx^2$. Use the graph to determine the values of A and B. Show relevant working.

(b) Evaluate f(1)

(c) Solve f(x) = 10 for x.

(d) Explain why the inverse of the function $f(x) = A + Bx^2$ is NOT a function?

(e) Restrict the domain of y = f(x) such that the inverse of f(x) is in fact a function.



(f) Perform the composition defined by the equation $f^{-1} \circ f(x)$ and explain the significance of this composition.

(g) Determine the average rate of change between: (i) $(0, f(0))$ and $(2, f(2))$ & (ii) $(-1, f(-1))$ and $(1, f(1))$

(h) Mr. S would like to specify various intervals in which the average rate of change is positive. Suggest two suggest intervals.

(i) In which interval is the function described as being an increasing function?

E. Problem 3 – New Context: Motion

The displacement, in meters, of an object, *t* seconds into its motion is given by the equation $s(t) = t^3 + 3t^2 + 2t,\ t \geq 0$. Find the average rate of change during:

a. the first second (include units in your answer and interpret the meaning of your answer);
b. the first four seconds (include units in your answer and interpret the meaning of your answer);
c. the interval when *t* = 1 to *t* = 1 + h
d. How could we predict the instantaneous rate of change of the object at t = 1 second?

F. Problem 4 – Consolidating Rational Functions, Asymptotes & Rates of Change

(j) Given the rational function $f(x) = \dfrac{2x}{8 + x^3}$, use the TI-84 to answer the following questions:

i. Determine the equation(s) of the horizontal asymptote(s).
ii. Describe the "end behaviour" of the function. Explain your reasonings.
iii. Evaluate the following "limits" ➔ $\lim_{x \to +\infty} f(x)$ as well as $\lim_{x \to -\infty} f(x)$. Explain your reasonings.
iv. Explain if/how/why the answers to the first three questions are all the same!!!!
v. Determine the equation(s) of the vertical asymptote(s).
vi. State the domain and range of $f(x) = \dfrac{2x}{8 + x^3}$.

The function $C(t) = \dfrac{2t}{8 + t^3}, t \geq 0$ can be used to model the concentration of a drug in the bloodstream of a patient, where *C*(*t*) is the concentration of the drug, measured in mg/ml and *t* is time measured in hours after the drug was injected into the patients bloodsteam

(k) Evaluate *C*(2) and explain what the meaning of this point.
(l) Determine the maximum concentration of the drug and at what time that occurs. Record the answer with appropriate units in your answer.
(m) During what time interval is the concentration of the drug increasing?
(n) Determine the average rate of change in the drug concentration: (include units with your numerical answers)

i. During the first hour;
ii. During the first 2 hours;
iii. During the period form t = 2 hours to t = 4 hours.

G. CONSOLIDATION:

Write a formula that would allow you to calculate an average rate of change. What are the limitations or restrictions of this formula?